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Theoretical Astrophysics
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Causal production of the electromagnetic energy flux in the Blandford-Znajek process

Toma & Takahara 2016, PTEP, 3E01 (arXiv:1605.03659)

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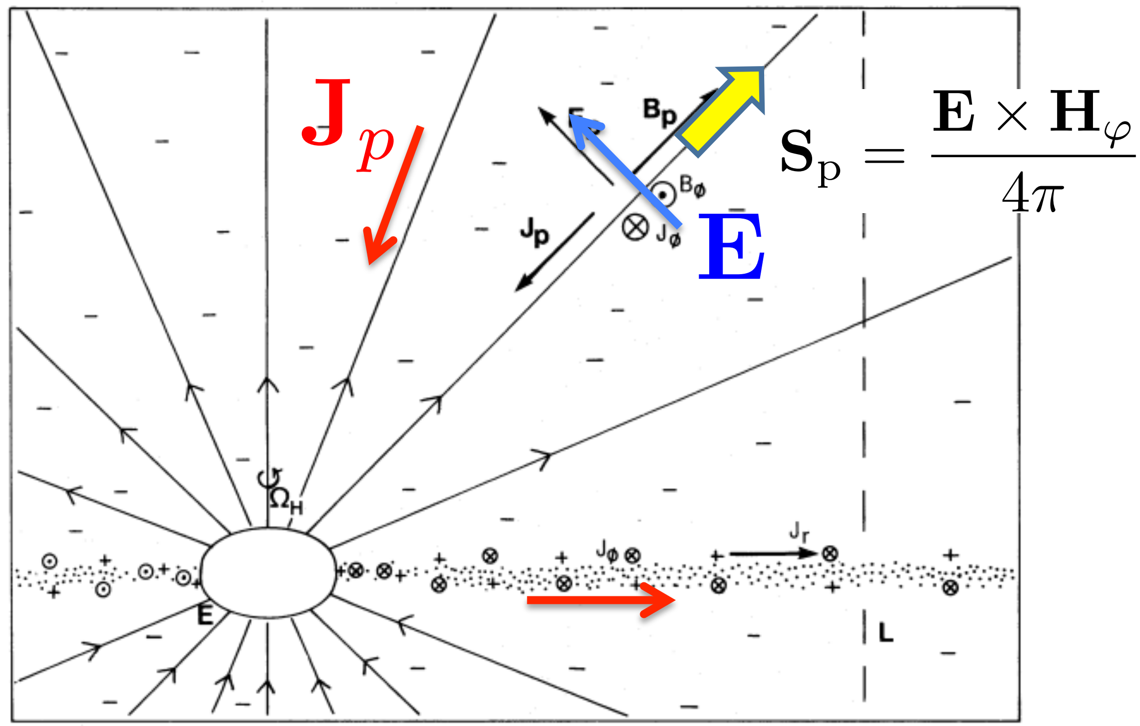
Blandford & Znajek (1977)

- Slowly rotating Kerr space-time

$$a = \frac{J}{M r_g c} \ll 1$$

- Steady, axisymmetric
- Split-monopole B field
- Force-free approximation (Electromagnetically dominated)

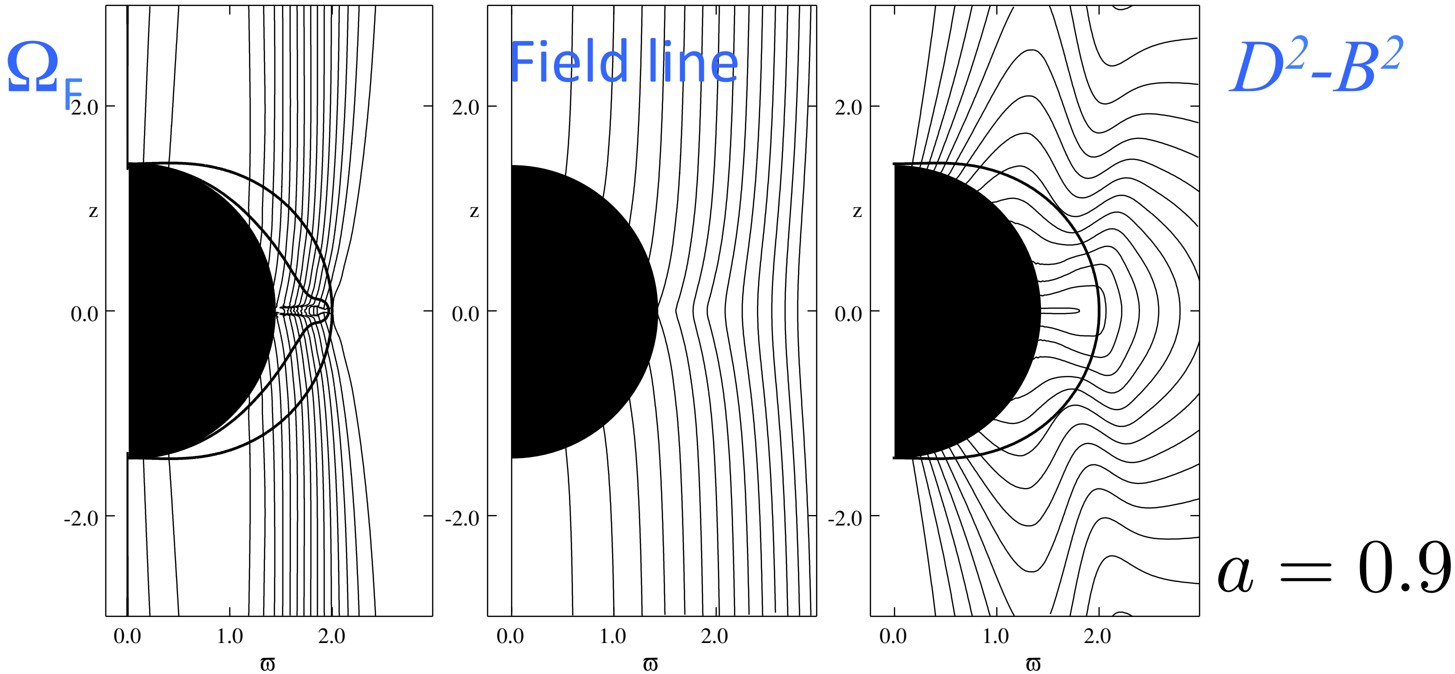


$$\mathbf{J}_\mathrm{p} \parallel \mathbf{B}_\mathrm{p} \qquad \mathbf{E} \perp \mathbf{B}$$

(see also Beskin & Zheltoukhov 2013)

BZ process with large BH spin a

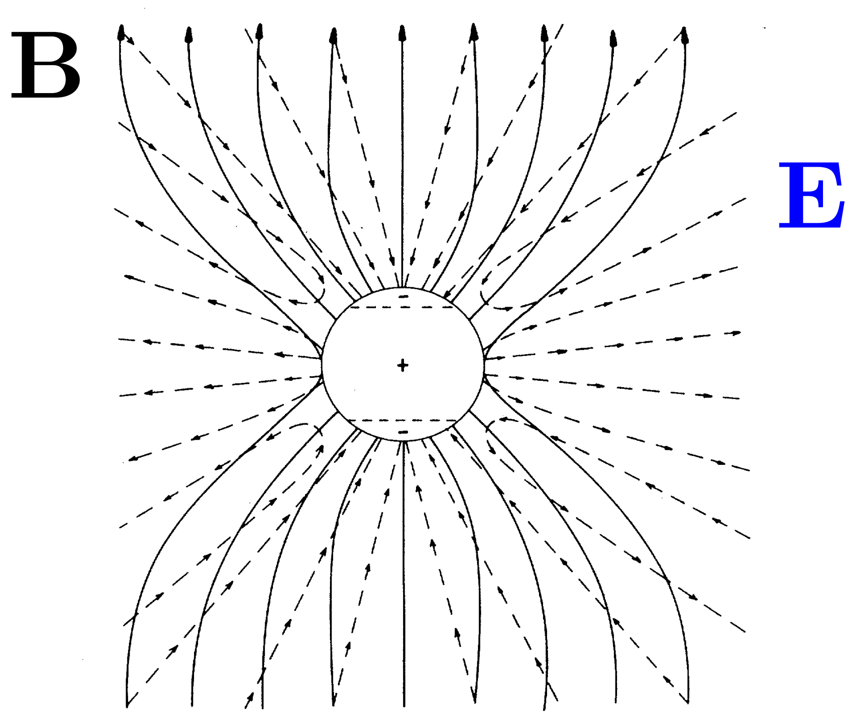
(Komissarov 2004)



- Many other FF/MHD numerical studies show BZ process works with large a. (e.g. Komissarov 01; McKinney 06; Barkov & Komissarov 09; Tchekhovskoy+ 11; Ruiz+ 12; Contopoulos+ 13)
- It is proved analytically that $E = 0$ cannot be maintained for open field lines (KT & Takahara 14)

But the detailed mechanism of flux production is still debated

Vacuum Solution

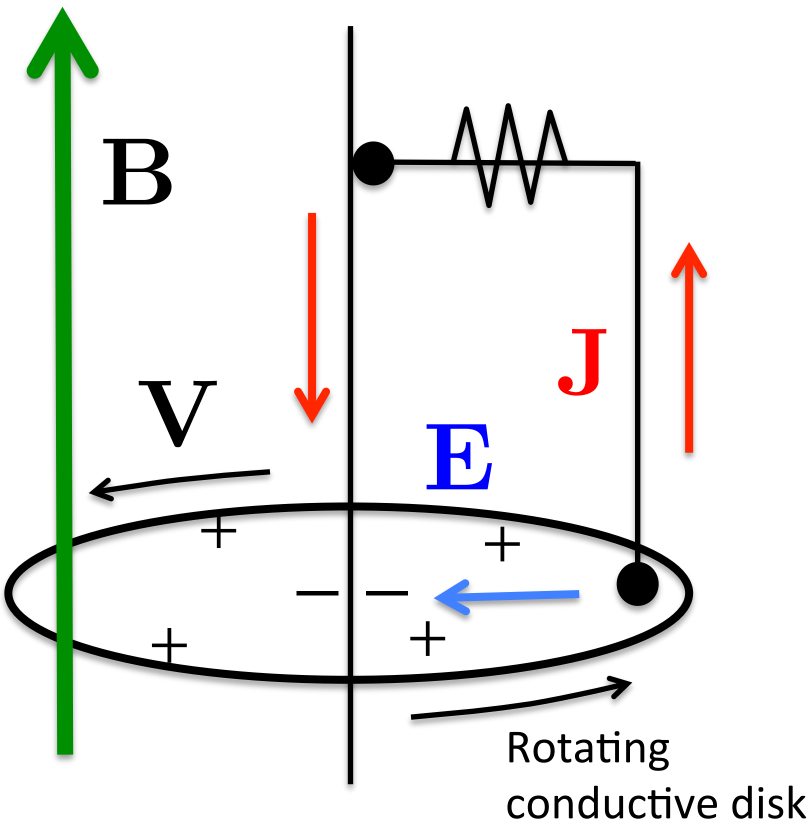


(Wald 1974; Punsly & Coroniti 1989)

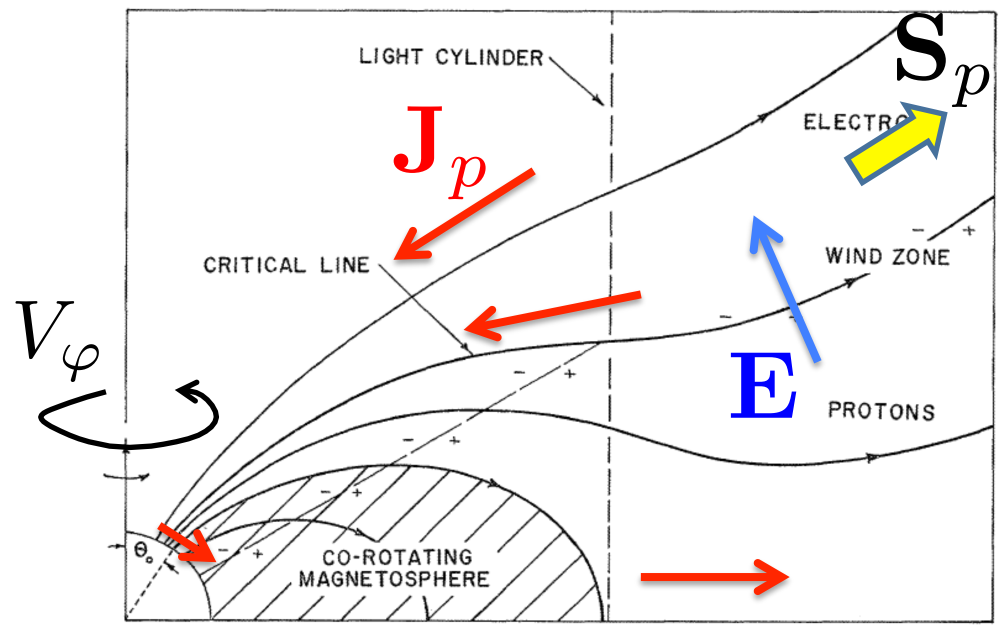
- Space-time rotation produces $\mathbb{E}$, but not B_ϕ
- B_ϕ requires J_p. What drives J_p ??

Unipolar induction

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B}$$



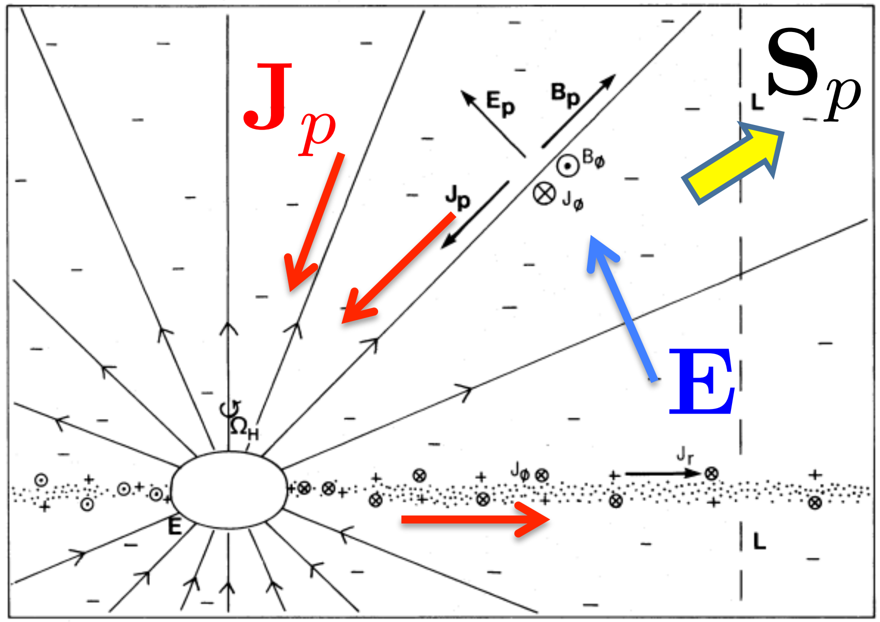
Pulsar winds (Goldreich & Julian 1969)



$$\nabla \cdot \mathbf{S}_\mathrm{p} = -\mathbf{E} \cdot \mathbf{J}_\mathrm{p}$$

Matter rotational energy reduced ⇐ Energy source!

There is no matter-dominated region in BZ process



(Blandford & Znajek 1977)

$$\boxed{\nabla \cdot \mathbf{S}_{\rm p} = 0}$$

Unipolar induction cannot work.

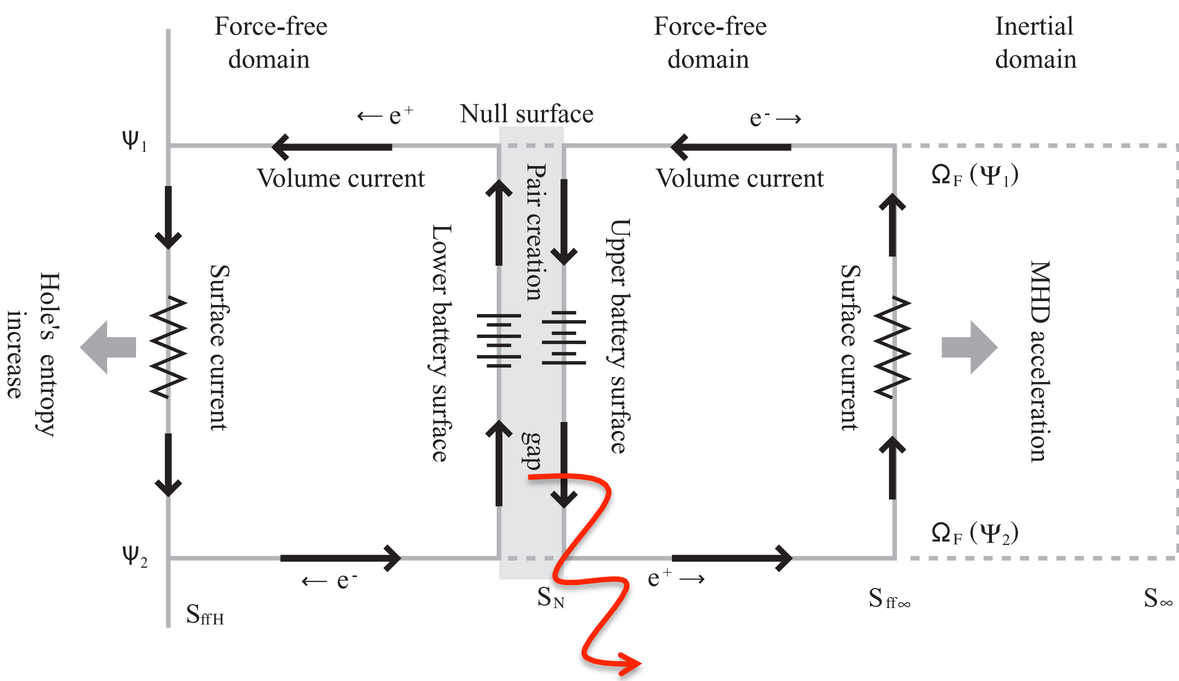
What drives J_p ??

Discussions so far

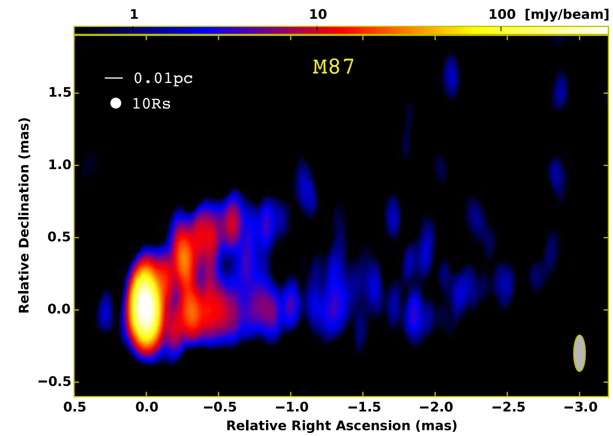
- Membrane paradigm
  - Horizon is assumed as a rotating conductor (Thorne et al. 1986; Penna et al. 2013)
  - Horizon is causally disconnected (Punsly & Coroniti 1989)
  - Current driving mechanism is unclear
- Negative electromagnetic energy inflow (Lasota+14; Koide & Baba 14)
  - $S_p = E\, H_\phi / 4\pi = \varepsilon\, v_p$ ($\varepsilon < 0$, $v_p < 0$)?
  - $\varepsilon = T_{EM}{}^0{}_0 > 0$ in Kerr-Schild coordinates (KT & Takahara 2016)
- MHD picture
  - v_p = particle velocity: $\varepsilon < 0$ even outside ergosphere (Takahashi +90)
  - Inertial drift current cannot produce all of S_p
  - No negative particle energy seen in MHD simulations (Komissarov 05)

Current driven in a pair creation gap ?

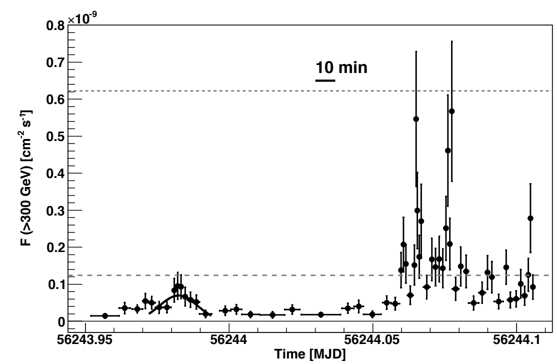
(Okamoto 2006)



This case could be relevant for the upcoming high-resolution radio observations and the observed high-variability gamma-rays.



M87 radio jet (Hada et al. 2016)



IC310 TeV gamma-rays (Aleksic et al. 2015, Science)

3+1 Electrodynamics

$$\boxed{ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(\beta^i dt + dx^i)(\beta^j dt + dx^j),}$$

$$E^\mu = F^{\mu\nu}\xi_\nu,\, H^\mu = -{}^*F^{\mu\nu}\xi_\nu$$

Fields in the coordinate basis

$$D^\mu = F^{\mu\nu}n_\nu,\, B^\mu = -{}^*F^{\mu\nu}n_\nu$$

Fields as measured by FIDOs/ZAMOs

$$\nabla \cdot \boldsymbol{B} = 0, \quad \partial_t \boldsymbol{B} + \nabla \times \boldsymbol{E} = 0,$$

$$\nabla \cdot \boldsymbol{D} = 4\pi\rho, \quad -\partial_t \boldsymbol{D} + \nabla \times \boldsymbol{H} = 4\pi \boldsymbol{J},$$

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B},$$

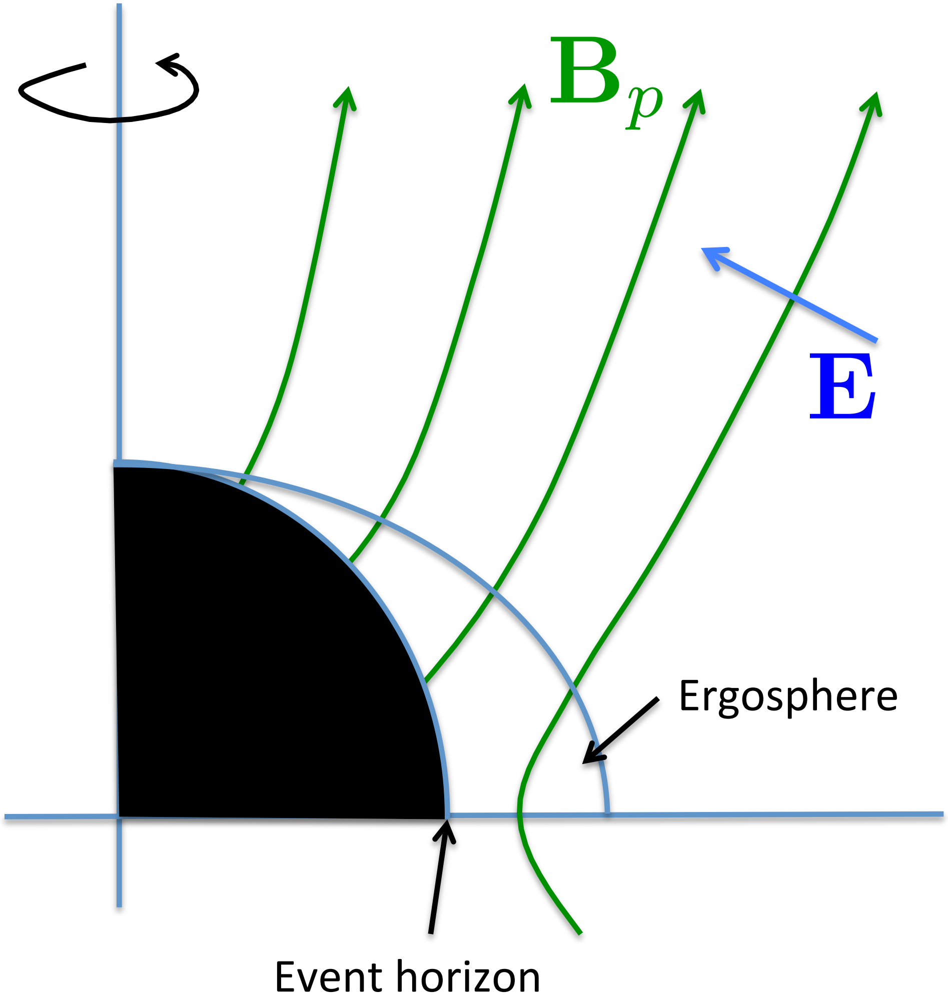
$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{D},$$

Electromagnetic energy equation

$$\partial_t \left[\underbrace{\frac{1}{8\pi}(\boldsymbol{E}\cdot\boldsymbol{D} + \boldsymbol{B}\cdot\boldsymbol{H})}_{\text{Energy density}}\right] + \nabla \cdot \left(\underbrace{\frac{1}{4\pi}\boldsymbol{E}\times\boldsymbol{H}}_{\text{Poynting flux}}\right) = -\boldsymbol{E}\cdot\boldsymbol{J},$$

(Landau & Lifshitz 1975; Komissarov 2004)

General conditions of magnetosphere



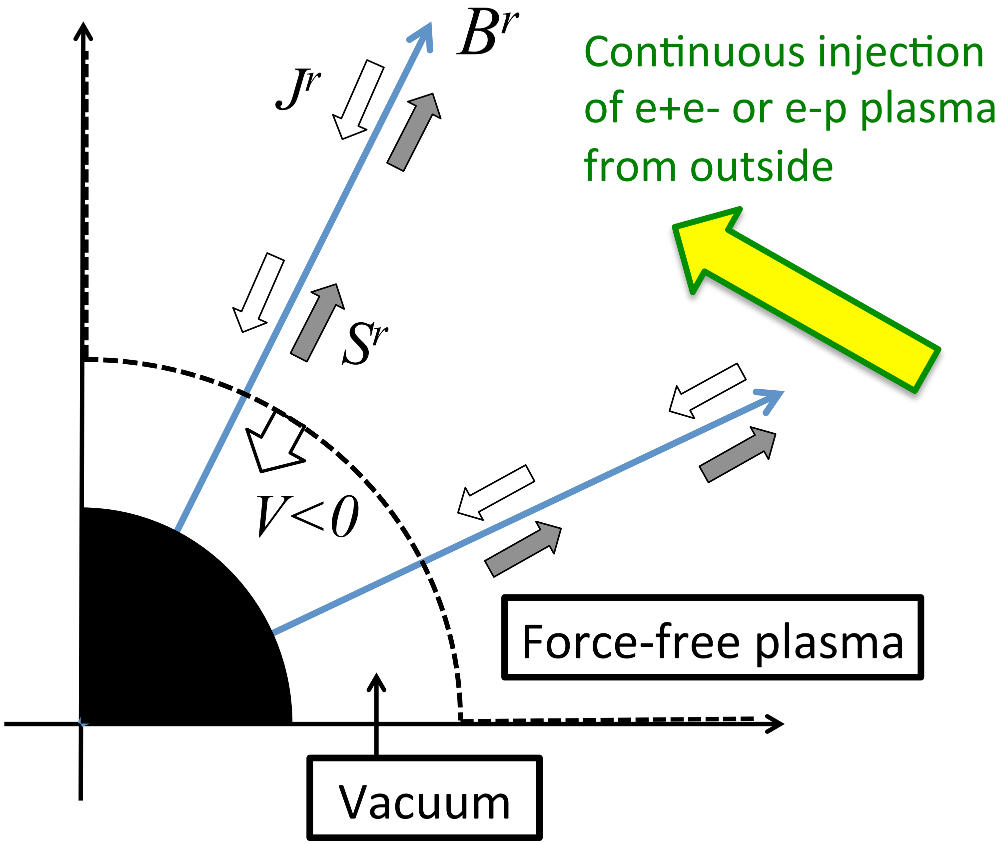
- Kerr spacetime with arbitrary spin a (fixed)
- Axisymmetric
- Poloidal B field (with arbitrary shape) threading the ergosphere
- Plasma with sufficient number density

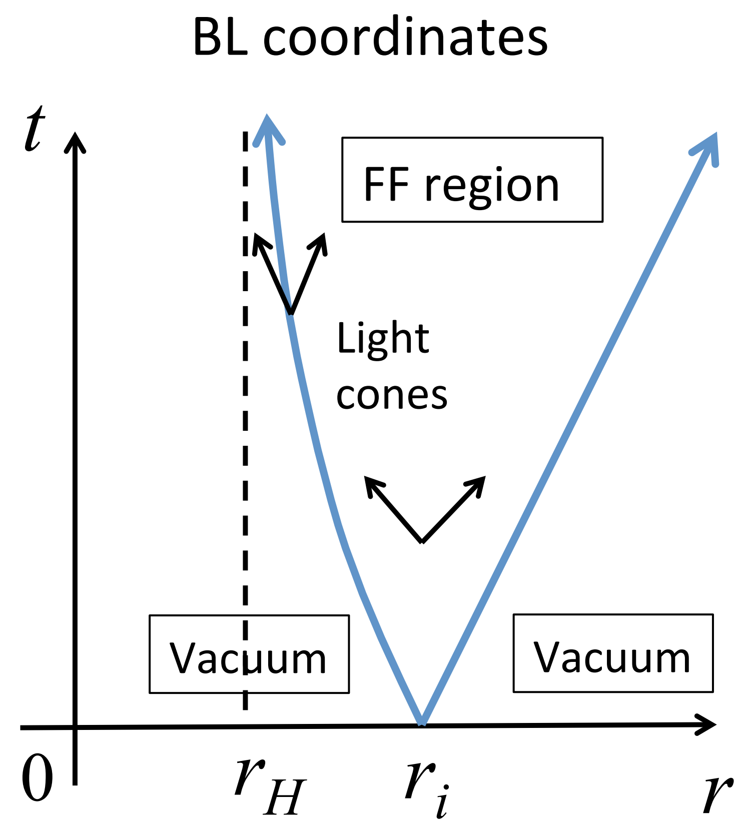
$$\mathbf{D} \cdot \mathbf{B} = 0$$

$$(\mathbf{E} \cdot \mathbf{B} = 0)$$

Process toward steady state

First consider a vacuum, and then begin injecting force-free plasma continuously between the two light surfaces

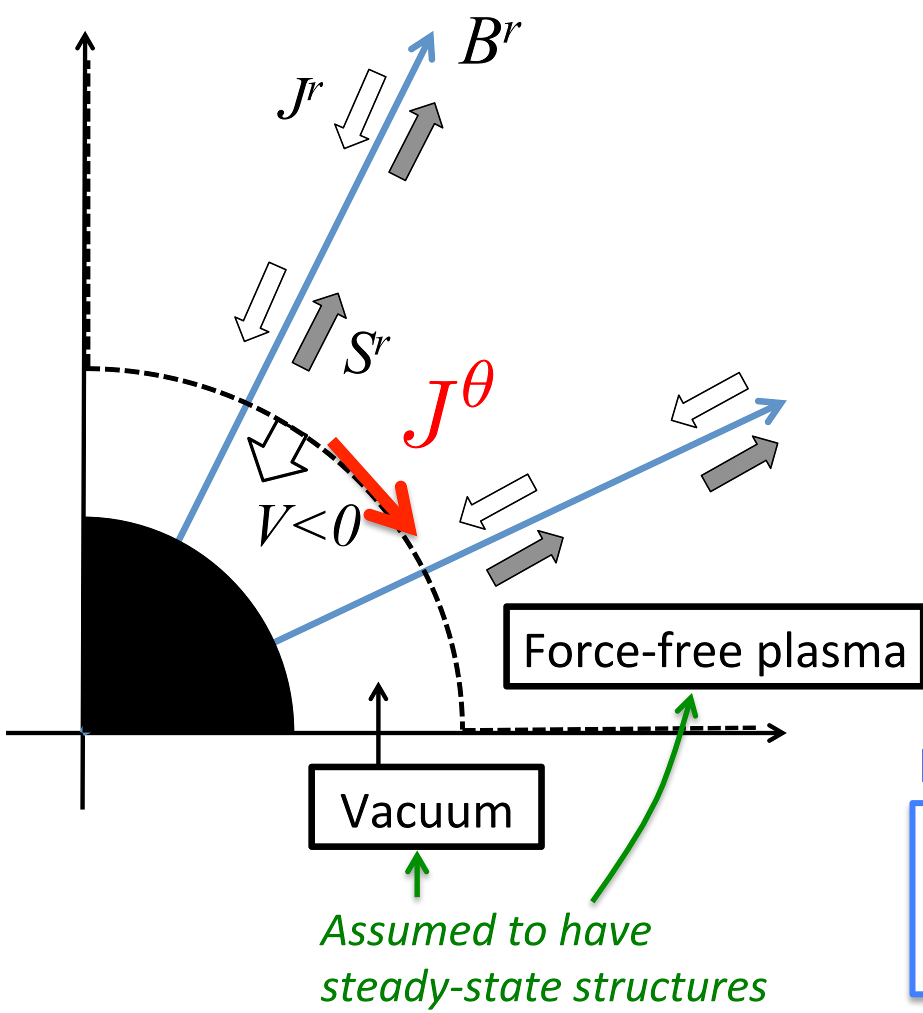




(KT & Takahara 2016)

Causal production of the flux

We derived junction conditions from Maxwell equations, and found current must cross field lines



$$\left\{ \begin{aligned} \eta^r &= \left.\frac{-D^r_{\rm vac}}{4\pi}\right|_{R=0} V, \\ V &= \frac{1}{\sqrt{\gamma}} \left.\frac{H^{\rm ff}_\varphi + 4\pi\sqrt{\gamma}\eta^\theta}{D^\theta_{\rm ff} - D^\theta_{\rm vac}}\right|_{R=0}. \\ V &= \frac{1}{\sqrt{\gamma}} \left.\frac{E^{\rm ff}_\theta - E^{\rm vac}_\theta}{B^\varphi_{\rm ff}}\right|_{R=0}, \end{aligned} \right.$$

Flux is produced at the ingoing boundary

$$\boxed{\nabla \cdot \mathbf{S}_p = -\partial_t e - \mathbf{E} \cdot \mathbf{J}_p}$$

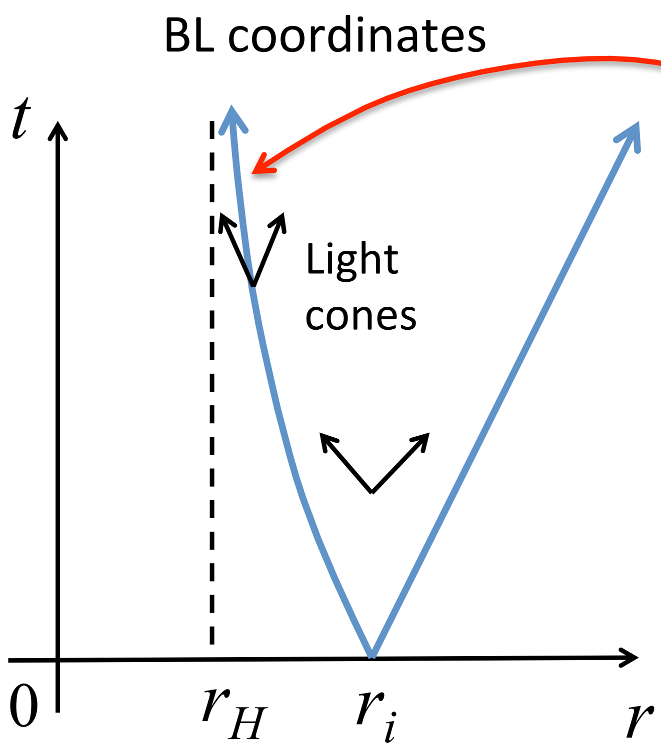
Steady State

$$\nabla \cdot \mathbf{L}_p = -\partial_t l - (\mathbf{J}_p \times \mathbf{B}_p) \cdot \mathbf{m}$$

$$\nabla \cdot \mathbf{S}_p = -\partial_t e - \mathbf{E} \cdot \mathbf{J}_p$$

$$\nabla \cdot \mathbf{L}_\mathrm{p} = 0$$

$$\nabla \cdot \mathbf{S}_\mathrm{p} = 0$$



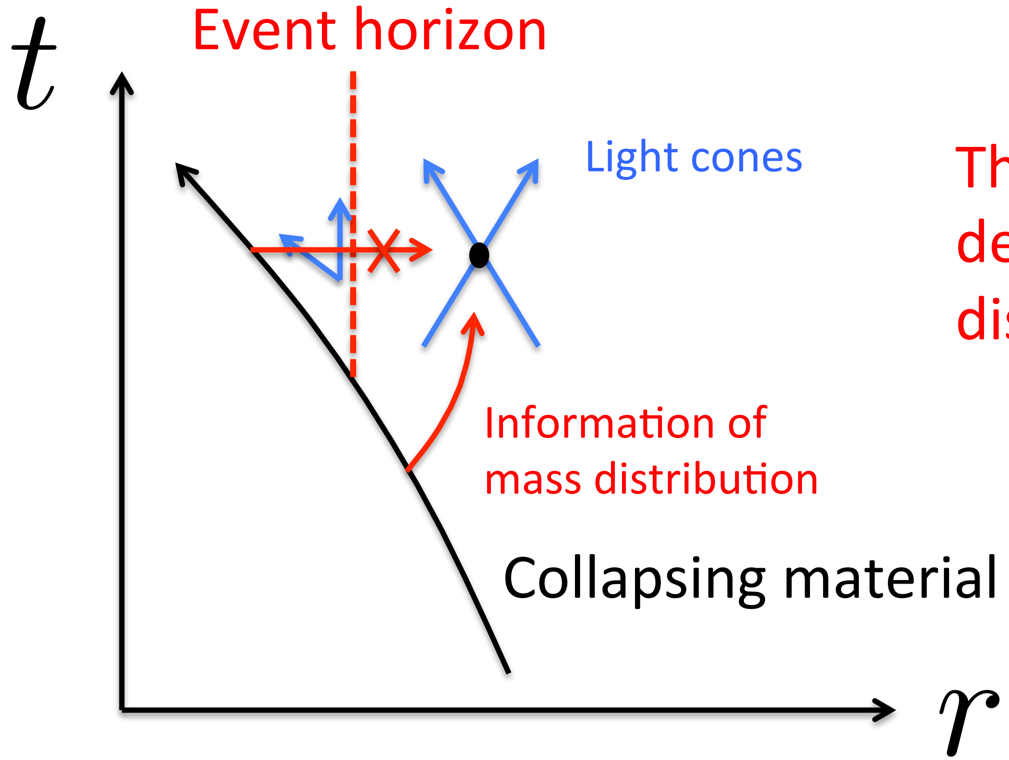
The boundary (AM/energy source) does not affect the exterior

- No electromagnetic sources are required in the steady state (partly because of no resistivity)

- BH decelerates directly by Poynting flux (different from mechanical Penrose process)

Origin of Schwarzschild spacetime

The source of Schwarzschild gravitational field is the mass inside the horizon, but the outside of horizon cannot know it



The spacetime metric is determined by the mass distribution at prior times

Conclusion

- The Poynting flux $S_p = E\,H_\phi/4\pi$ in the BZ process consists of the steady current flows in the electric potential differences
- The current driving (S_p production) mechanism can be discussed only in the time-dependent state towards steady state, like the mass source of a BH
- In the steady state, S_p needs no electromagnetic source. The steady currents can keep flowing in the ideal MHD condition. No gap is needed. The BH rotational energy is reduced directly by S_p without being mediated by the negative energies.
- Our argument is based on some assumptions. Detailed plasma simulations are needed to validate it

Back-up slides

Negative electromagnetic energy?

$$S_{\rm p} = e v_{\rm p} > 0$$ for $e < 0$ & $v_{\rm p} < 0$ (Lasota et al. 2014; Kiode & Baba 2014)

- Electromagnetic energy density e in the Boyer-Lindquist coordinates can be negative for $\Omega < \Omega_{\rm F}$

$$-\alpha T^t_t = \ e = \frac{1}{8\pi\alpha}\left[\alpha^2 B^2 + \gamma_{\varphi\varphi}(\Omega_{\rm F}^2 - \Omega^2)(B^\theta B_\theta + B^r B_r)\right].$$

- But $v_{\rm p}$ is not defined. The concept of advection of steady field is ambiguous.
- We showed $e > 0$ in the Kerr-Schild coordinates

$$S_p = \Omega_{\rm F}\frac{-H_\varphi}{4\pi}$$

(KT & Takahara 2016)

MHD model

Energy flux density

Bernoulli constant

$$S_\mathrm{p} = 4\pi\rho c^2\Gamma v_\mathrm{p}\mathcal{E} > 0 \quad \text{for} \quad v_\mathrm{p} < 0, \quad \mathcal{E} < 0$$

(Takahashi et al. 1990)

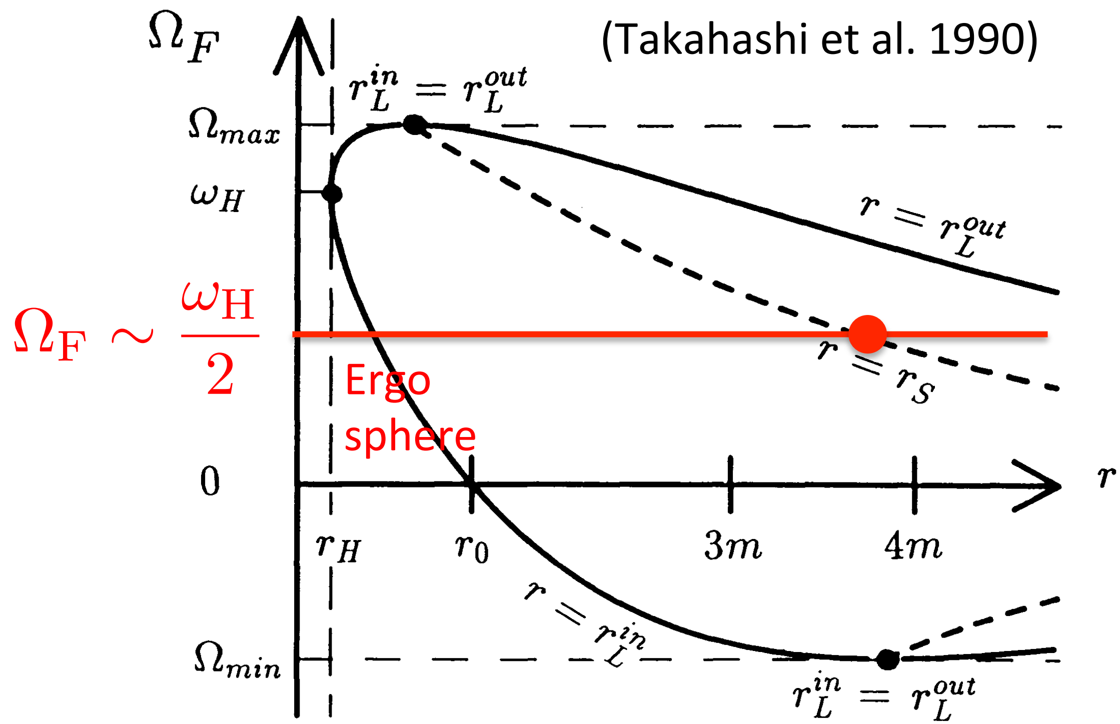


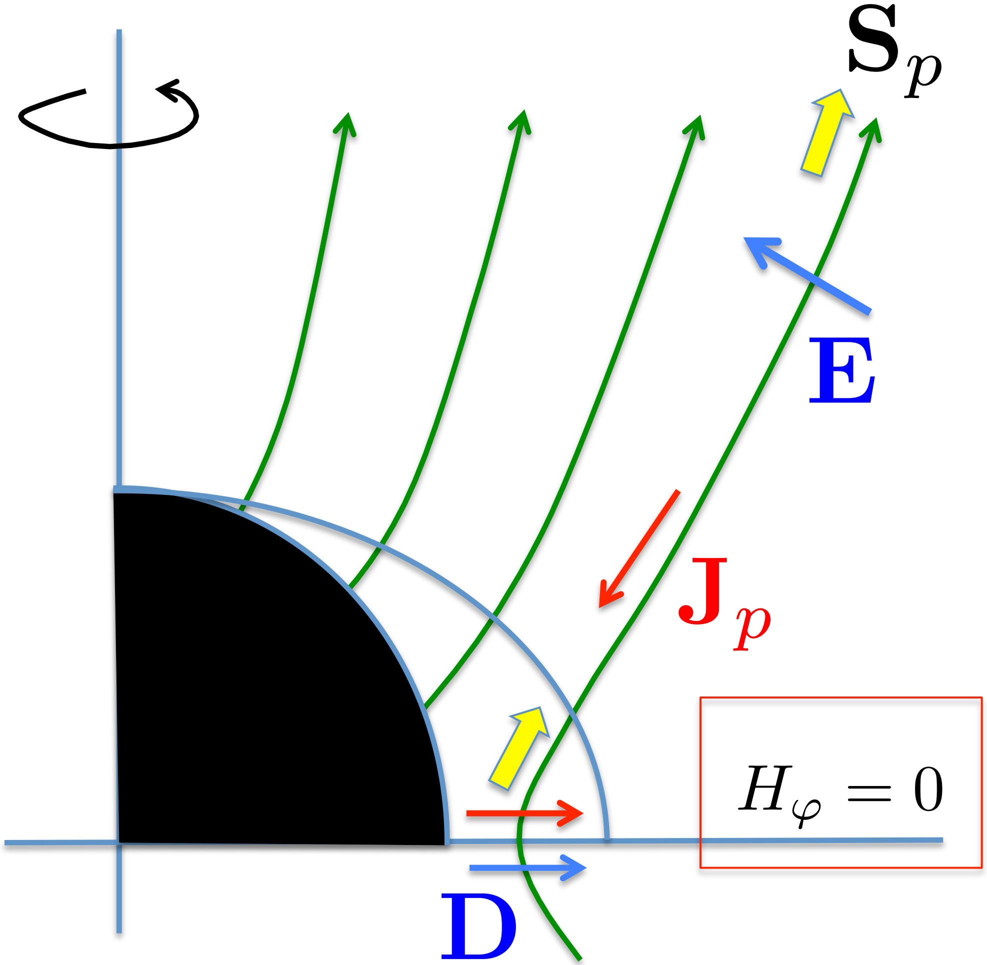
FIG. 1.—Positions of the light surfaces $r = r_L^{\mathrm{in}}$, r_L^{out} (*solid lines*) and the separation point $r = r_S$ (*broken lines*) for a monopole geometry in the equatorial plane with $a = 0.8m$. These points are determined by Ω_F.

Separation surface may be located outside the ergosphere.

(Komissarov 2009)

- Cross-field (inertial drift) currents cannot produce all of S_p
- MHD simulations show the steady state without negative particle energy (Komissarov 2005)

Field lines threading equatorial plane



- $D^2 > B^2$ possible, creating AM flux (H_ϕ) & Poynting flux

$$\nabla \cdot \mathbf{L}_p = -(\mathbf{J}_p \times \mathbf{B}_p) \cdot \mathbf{m}$$

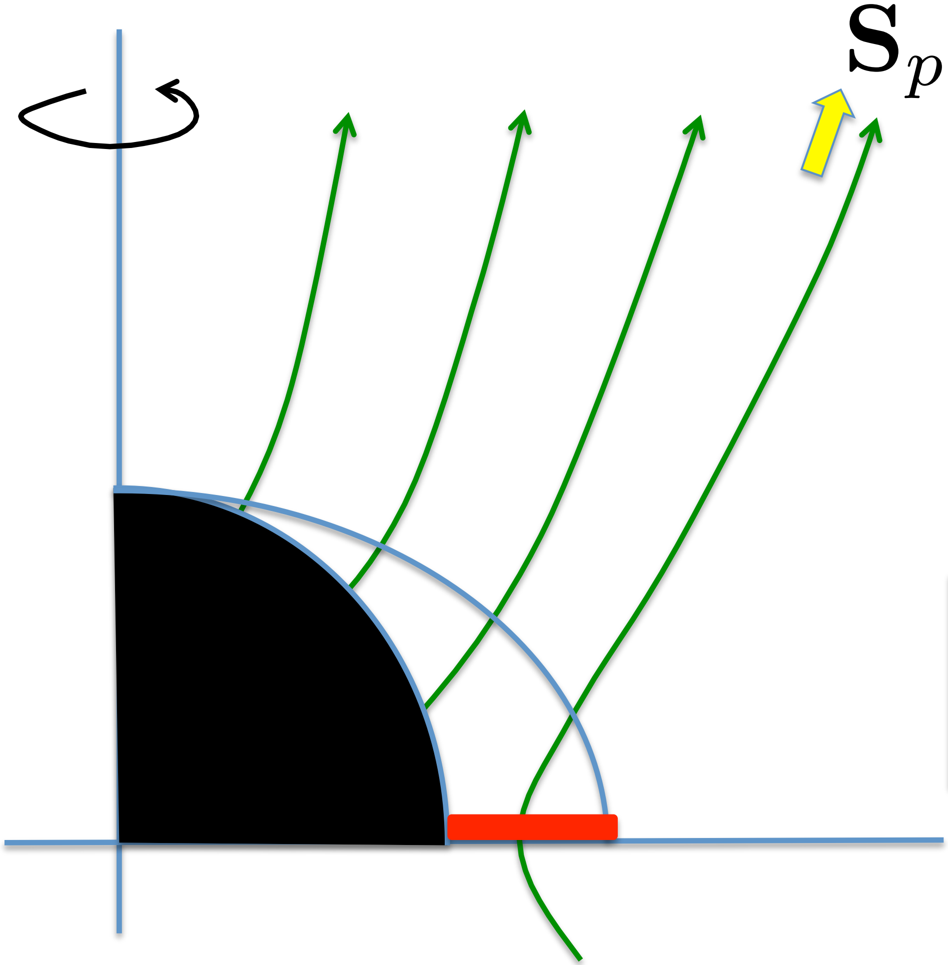
$$\nabla \cdot \mathbf{S}_p = -\mathbf{E} \cdot \mathbf{J}_p$$

- For $D^2 \sim B^2$, particles are strongly accelerated in direction of $-\phi$, obtaining negative energies

- Analogous to the mechanical Penrose process

(KT & Takahara 2014, 2016)

Inflow of negative-energy particles



$$\boxed{-U_t < 0, \quad U^r < 0}$$

$$\partial_r \sqrt{\gamma}(-\alpha \rho_{\rm m} U_t U^r) = \mathbf{E} \cdot \mathbf{J}_{\rm p} < 0$$

Znajek condition

$$H_\varphi = -\alpha \sqrt{\frac{\gamma_{\varphi\varphi}}{\gamma_{\theta\theta}}} D_\theta$$

BL coordinates

- Ohm's law for the current flowing on the membrane (Thorne et al. 1986 "Membrane Paradigm")

- Rather, it should be interpreted as displacement current (see also Punsly 2008)

$$H_\varphi^{\rm ff} = \sqrt{\gamma}(D_{\rm ff}^\theta - D_{\rm vac}^\theta)V - 4\pi\sqrt{\gamma}\eta^\theta$$

$$V = \frac{\pm\alpha}{\sqrt{\gamma_{rr}}}\sqrt{1 + \frac{4\pi\sqrt{\gamma}\eta^\theta}{H_\varphi^{\rm ff}}}.$$

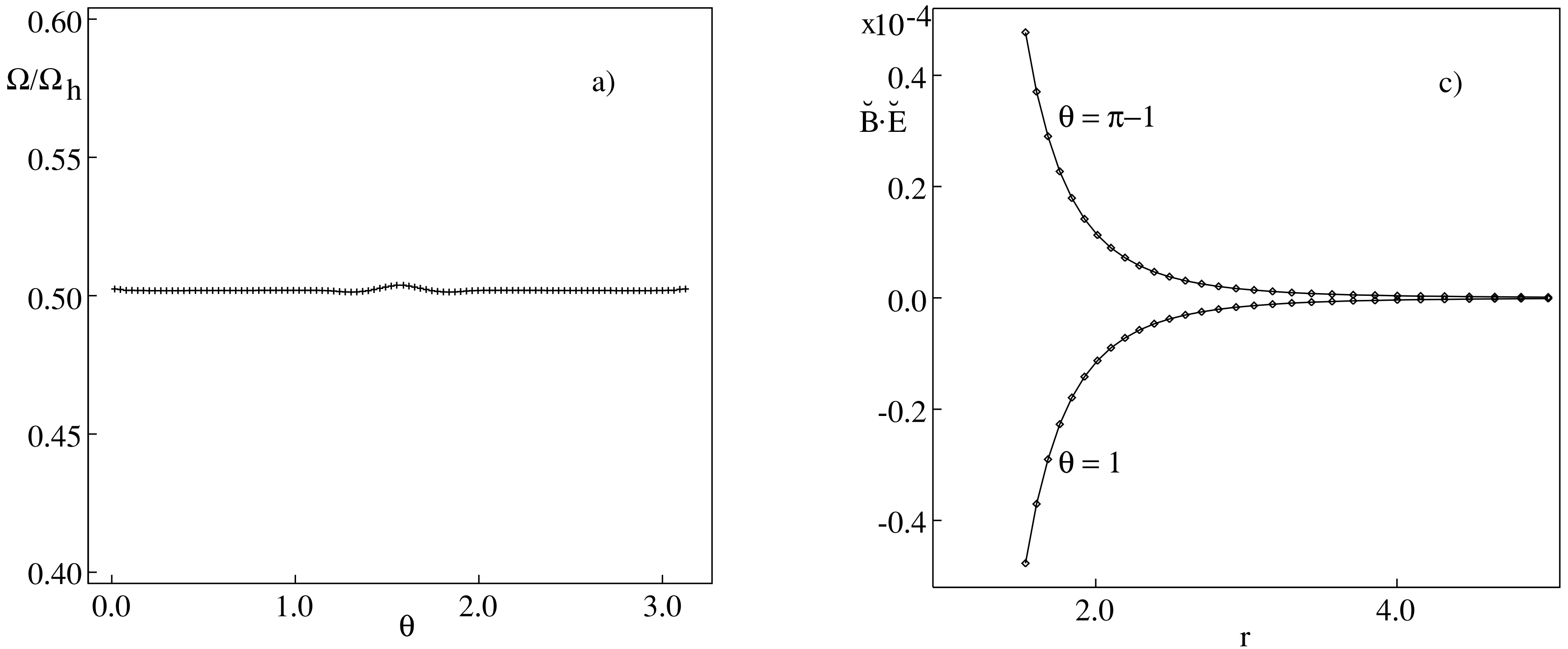
$$\eta^\theta \to 0$$

$$\alpha D_{\rm vac}^\theta \to 0$$

Resistive FF simulation results

Monopole solution with a = 0.1

(Komissarov 2004)



$$\mathbf{D} \cdot \mathbf{B} \neq 0$$

We consider that a small field-aligned electric field may appear in numerical simulations and in reality with small resistivity

2-fluid calculations

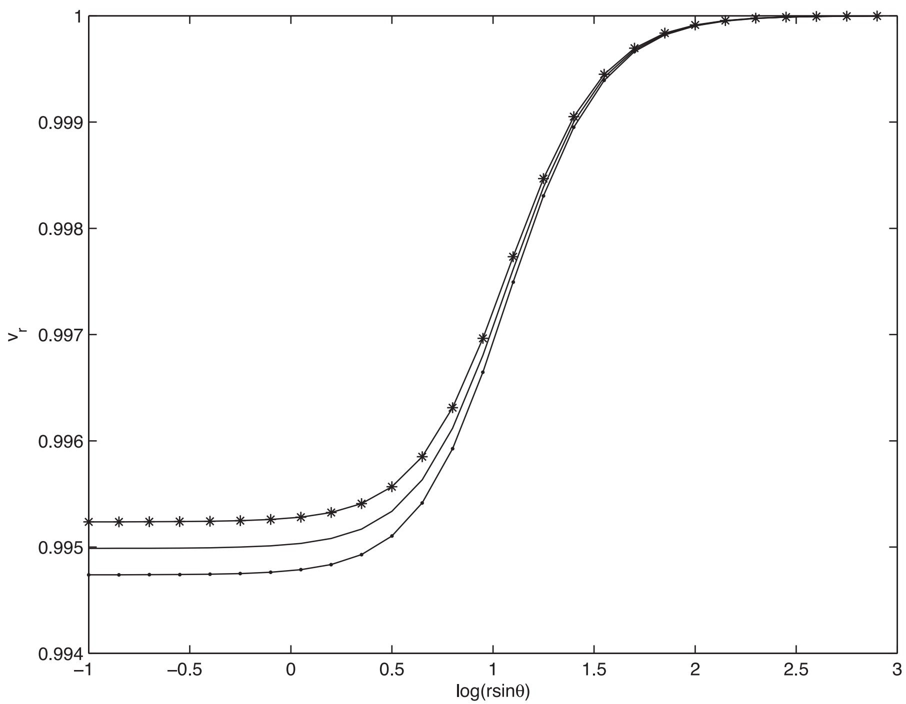
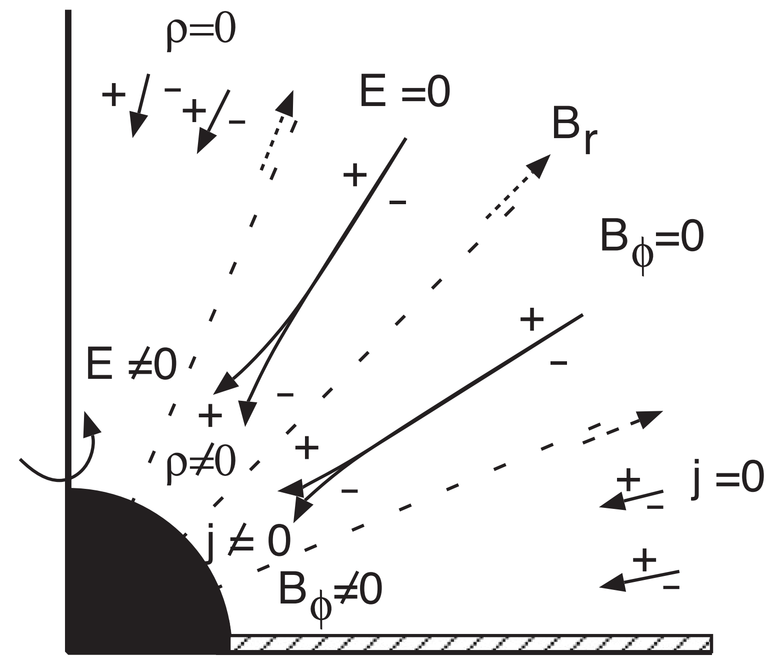


Figure 1. Radial velocity component for electrons (asterisks) and positrons (points) as a function of the axial distance; $\varepsilon = 0.1$, $\gamma_i = 10$. The solid line without markers shows the quantity w_0 determined by equation (25).



(Kojima 2015)

(Petrova 2015)

These 2-fluid analyses show the global violation of

$$\mathbf{E} \cdot \mathbf{B} = 0$$

Origin of Electromotive Force

$$\boxed{\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B},}$$

If $E=0$, $H_\phi=\alpha B_\phi=0$ (No ang. mom. or Poynting flux) along a field line,

$$\mathbf{D} = -\frac{1}{\alpha}\beta \times \mathbf{B}_p \quad \Rightarrow \quad D^2 > B^2 \text{ for } \alpha^2 < \beta^2$$

(in the ergosphere)

Then the force-free is violated, and the strong D field drives J_p across B_p ($H_\phi \neq 0$), weakening D ($E \neq 0$).

$\boxed{\text{The origin of the electromotive force is ascribed to the ergosphere.}}$

(KT & Takahara 2014, MNRAS; see also Komissarov 2004; 2009)

Blandford & Znajek (1977)

- Kerr space-time
- Steady, axisymmetric
- Slowly rotating BH

$$a = \frac{J}{M r_g c} \ll 1$$

- Split-monopole B field

$$B^r \sqrt{\gamma} = \text{const.}$$

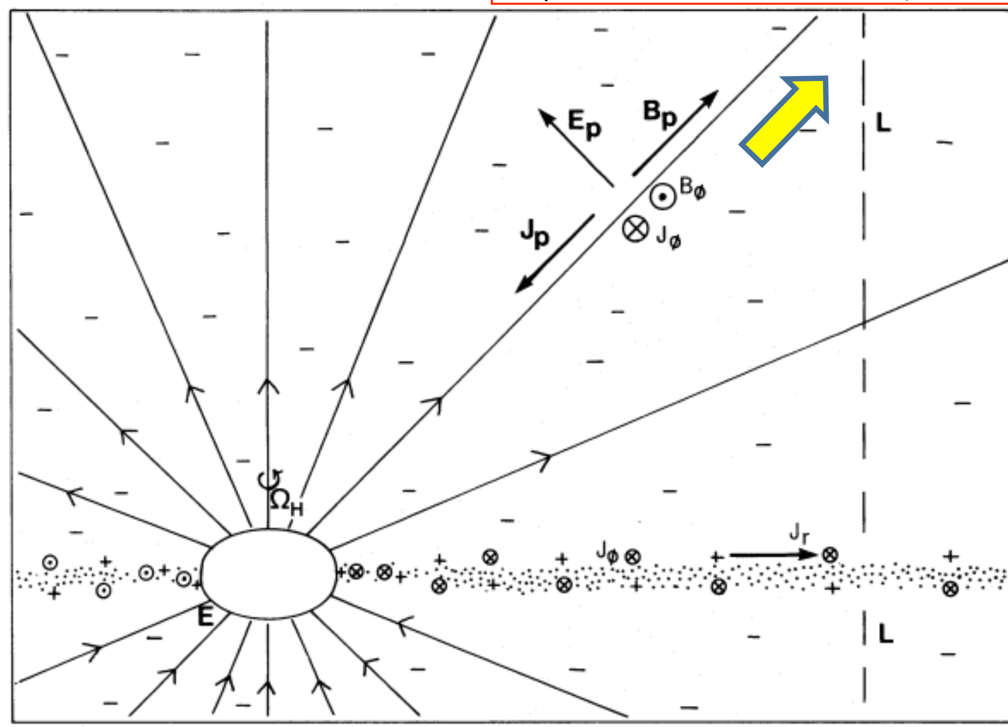
- Force-free approximation (Electromagnetically dom.)

$$H_\varphi = \text{const.}$$

$$\mathbf{E} = -\Omega_{\mathrm{F}} \mathbf{e}_\varphi \times \mathbf{B}$$

"Field line angular velocity"

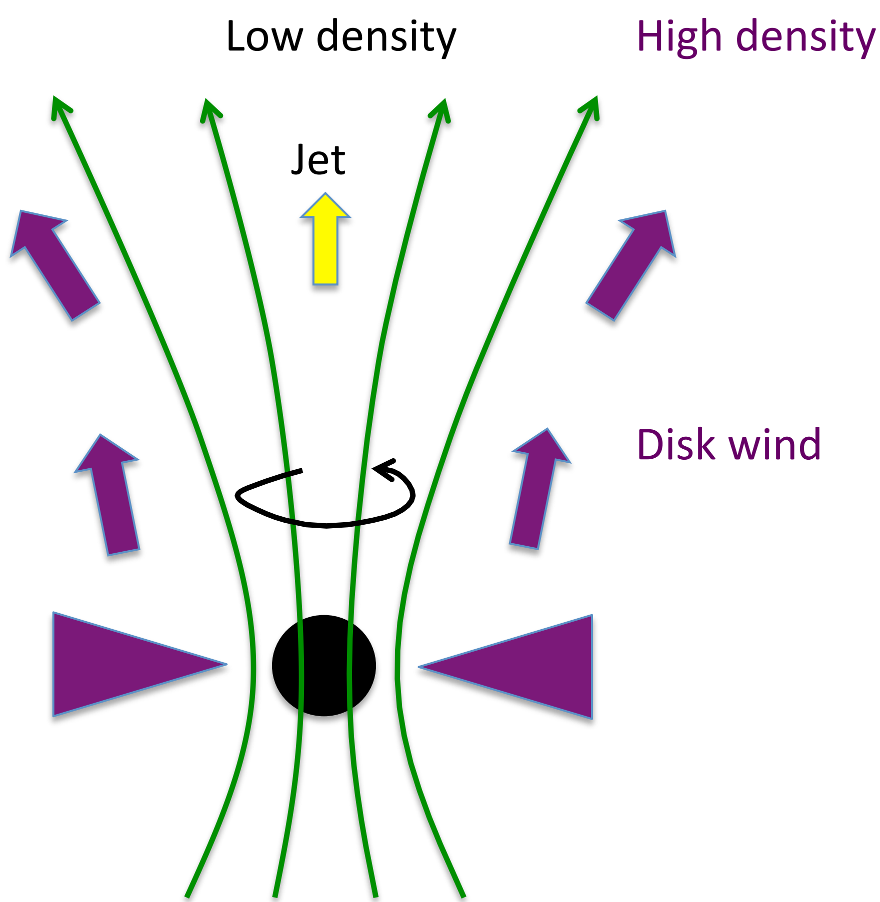
Condition at infinity $\boxed{H_\varphi = -2\pi\Omega_{\mathrm{F}} B^r \sqrt{\gamma} \sin\theta}$



$\boxed{H_\varphi = 2\pi(\Omega_{\mathrm{F}} - \Omega_{\mathrm{H}}) B^r \sqrt{\gamma} \sin\theta}$ At event horizon

$$\boxed{\Omega_{\mathrm{F}} = \Omega_{\mathrm{H}}/2 + O(a^3)}$$

Promising Scenario



Consistent with the radio data of M87 jet
(e.g. Asada+ 14; Kino+ 15)

- Energy injection into dilute region above BH → Relativistic speed
- Steady extraction of BH rotational energy (Blandford & Znajek 1977) → Poynting-dom jet

- Origin of jet matter debated (see KT & Takahara 2012)
- Matter acceleration by Lorentz force
- Collimation by external pressure (many literatures; see Lyubarsky 2009)