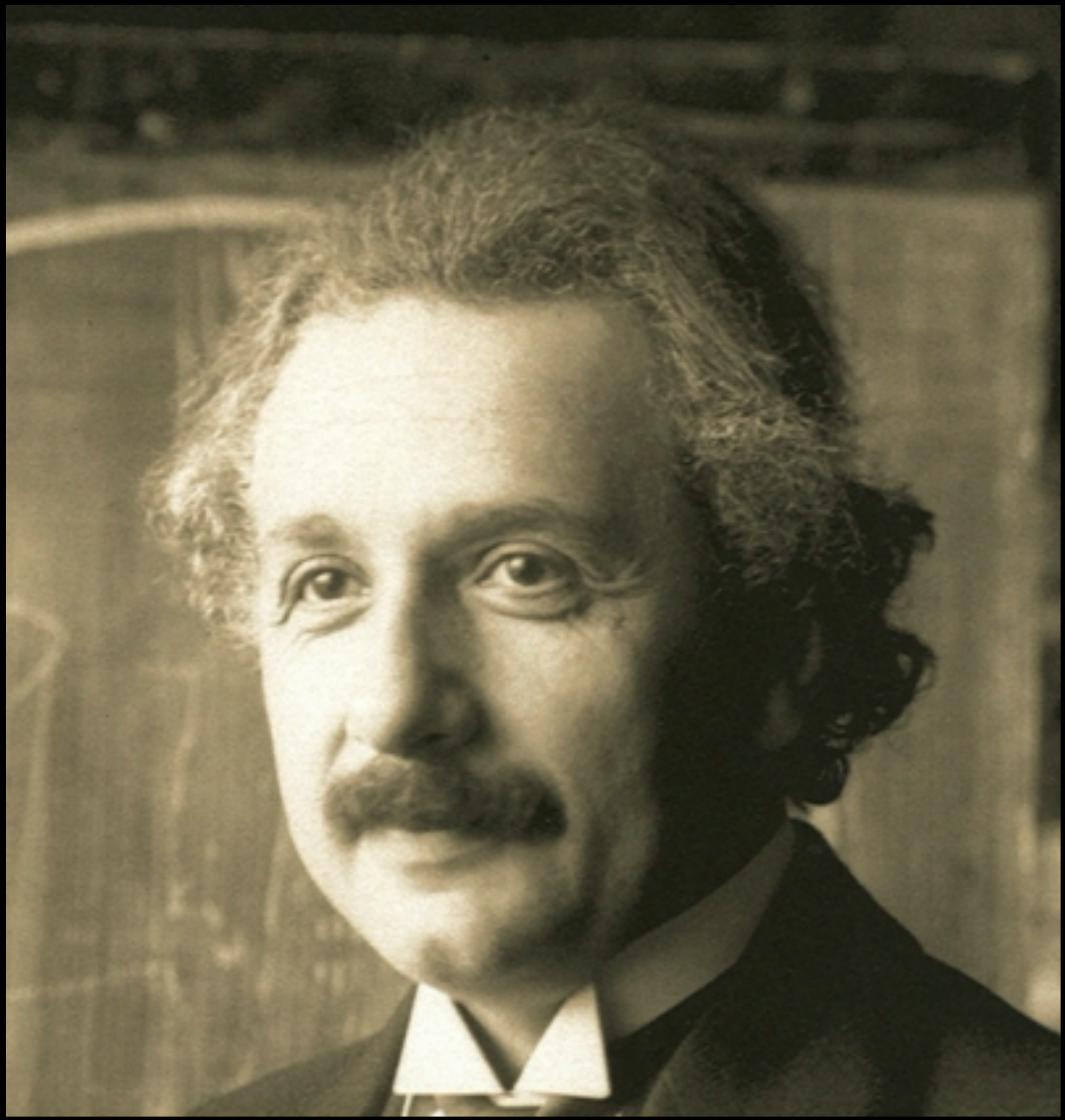
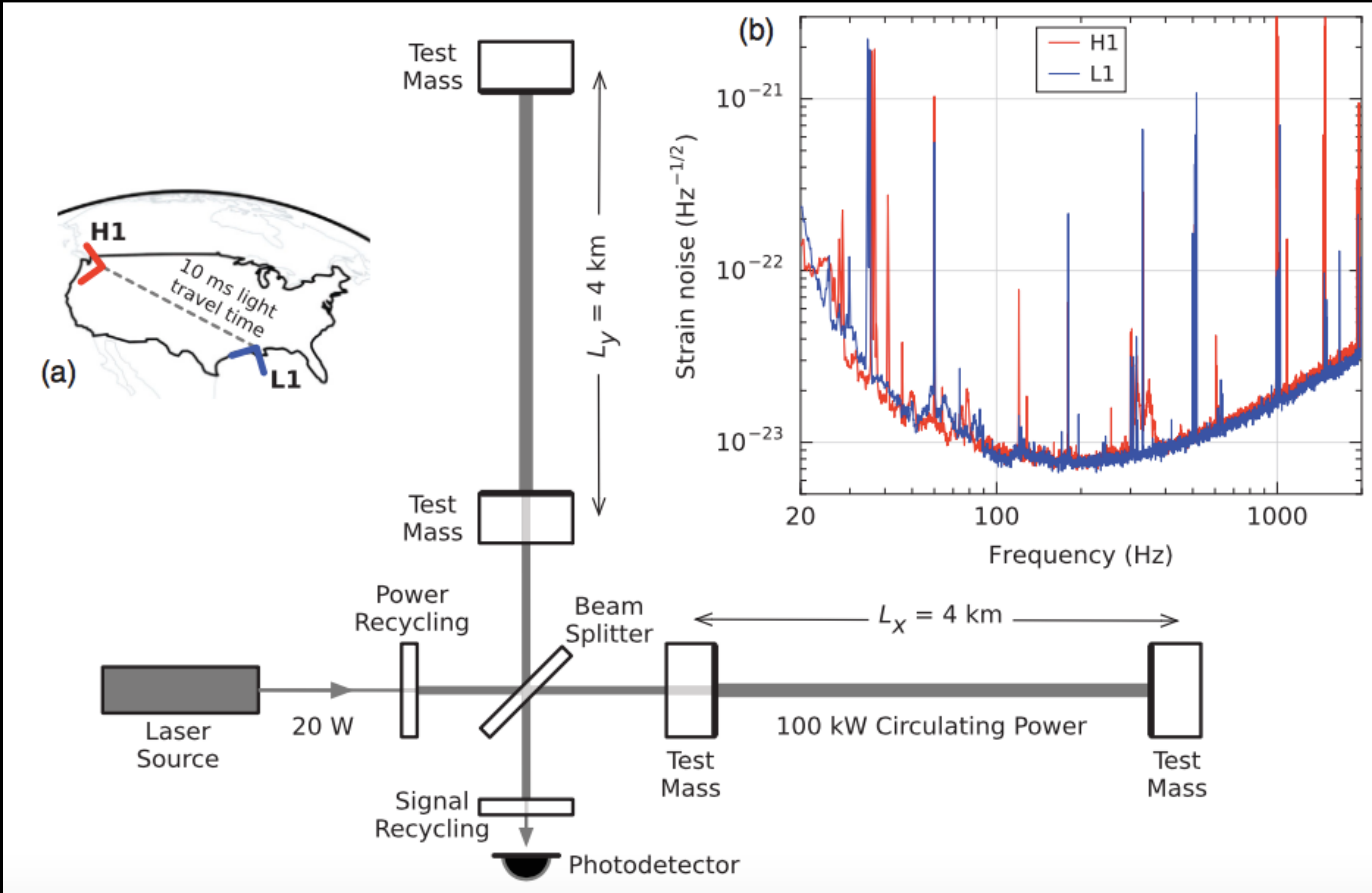
How to measure black holes: numerical relativity and gravitational waves

Mark Hannam
Cardiff University

IAU Symposium
Ljubljana, September 15, 2016



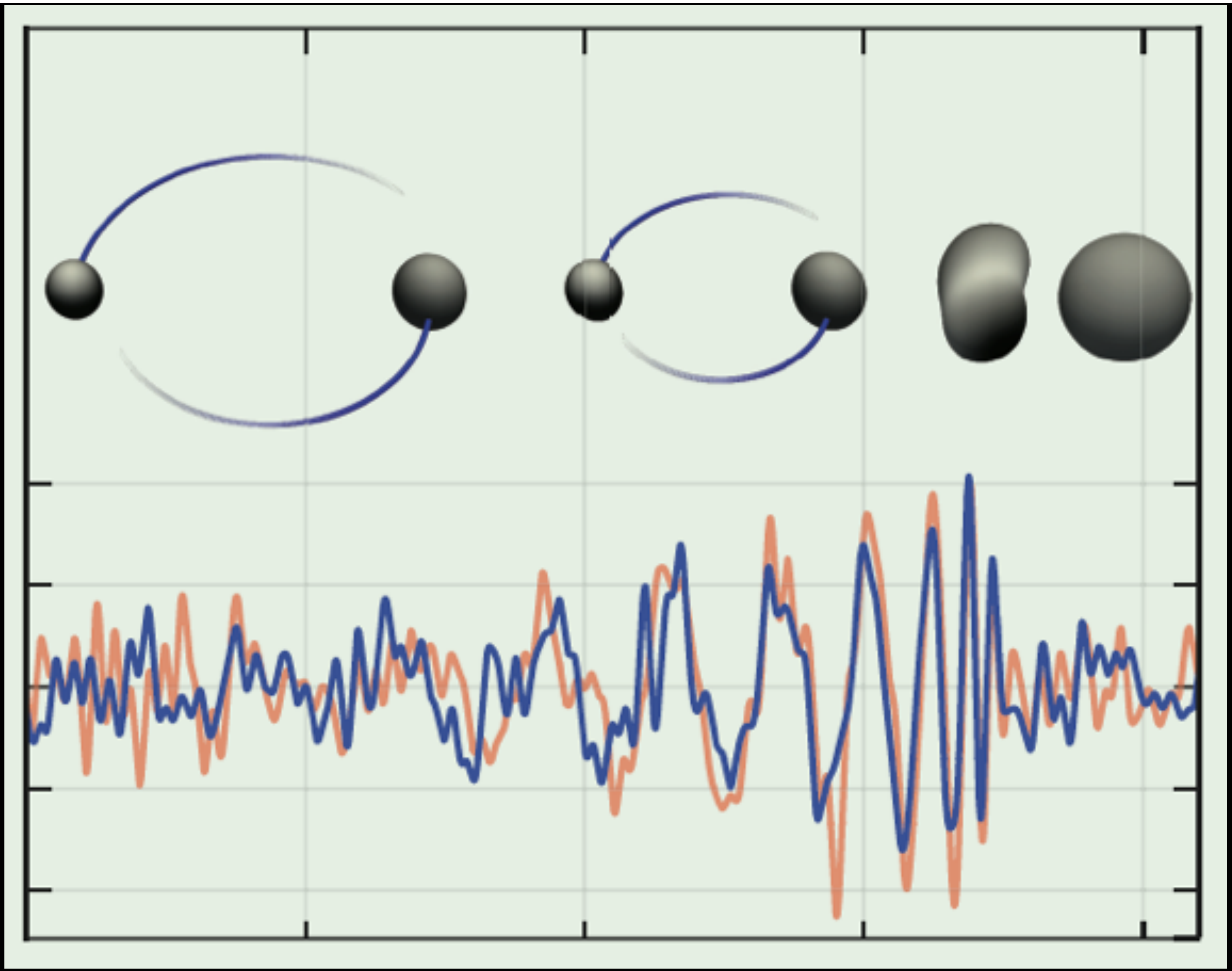


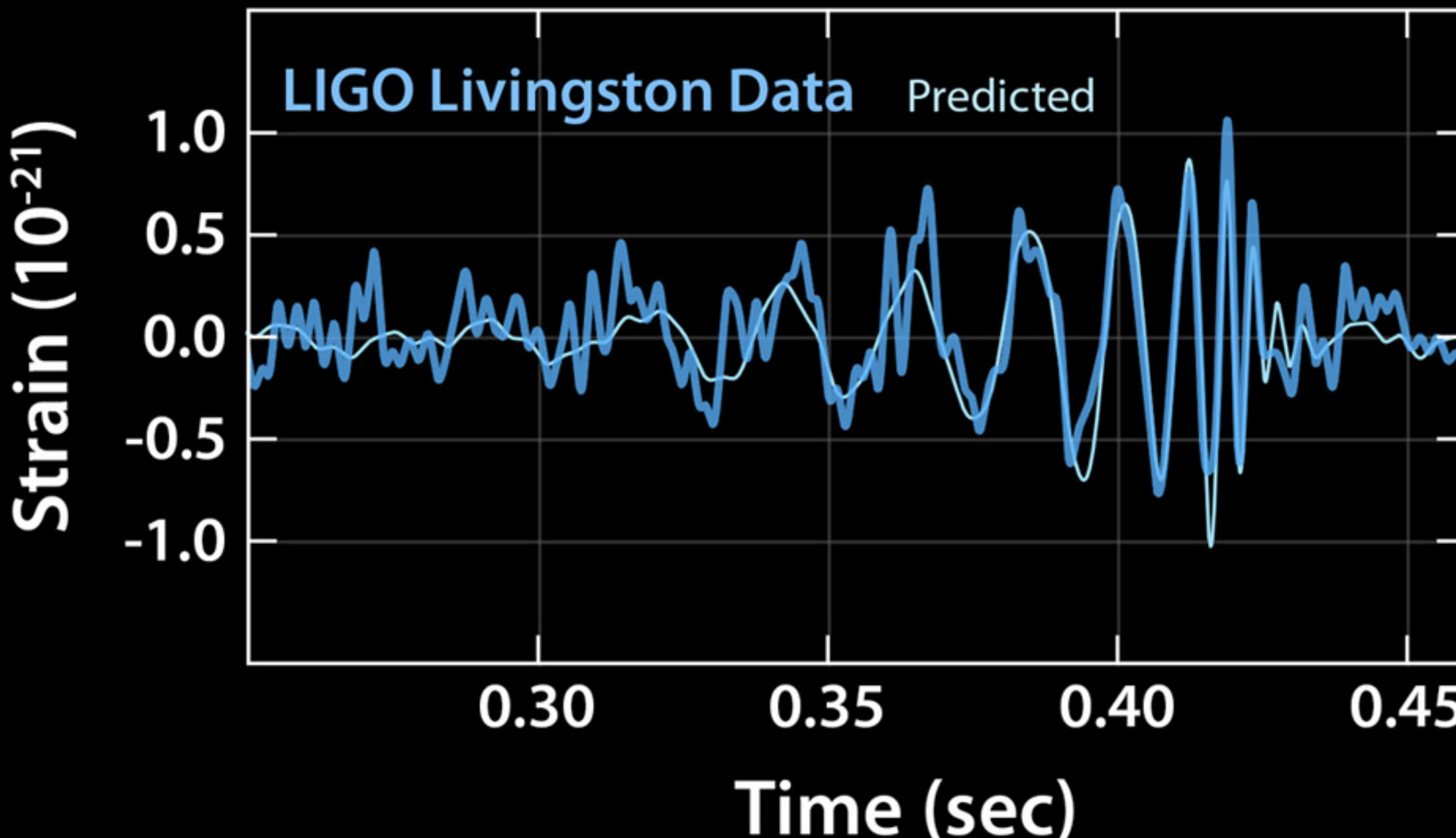
(a)
H1
L1
10 ms light travel time
(b)
H1
L1
Strain noise (Hz$^{-1/2}$)
10^{-21}
10^{-22}
10^{-23}
20
100
1000
Frequency (Hz)
Test Mass
Test Mass
L_y = 4 km
Power Recycling
Beam Splitter
L_x = 4 km
Laser Source
20 W
Test Mass
100 kW Circulating Power
Test Mass
Signal Recycling
Photodetector

Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

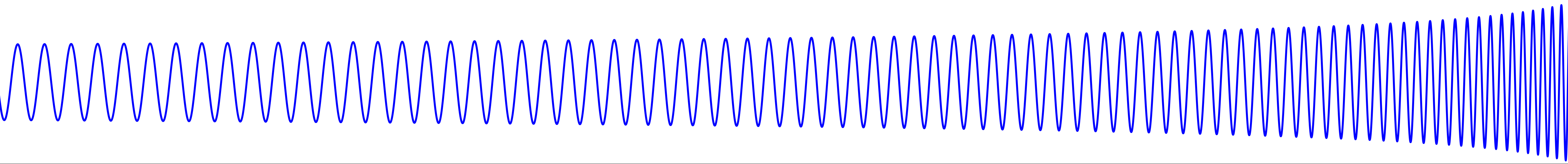



LIGO Livingston Data
Predicted
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)

Signal modelling

GWs take energy from system, orbit decays:

$$\frac{dE}{dt} \sim -v(t)^{10} \sim -\Omega(t)^{10/3}$$



Precision inspiral waveforms: post-Newtonian calculations (including effective-one-body [EOB] resummation)

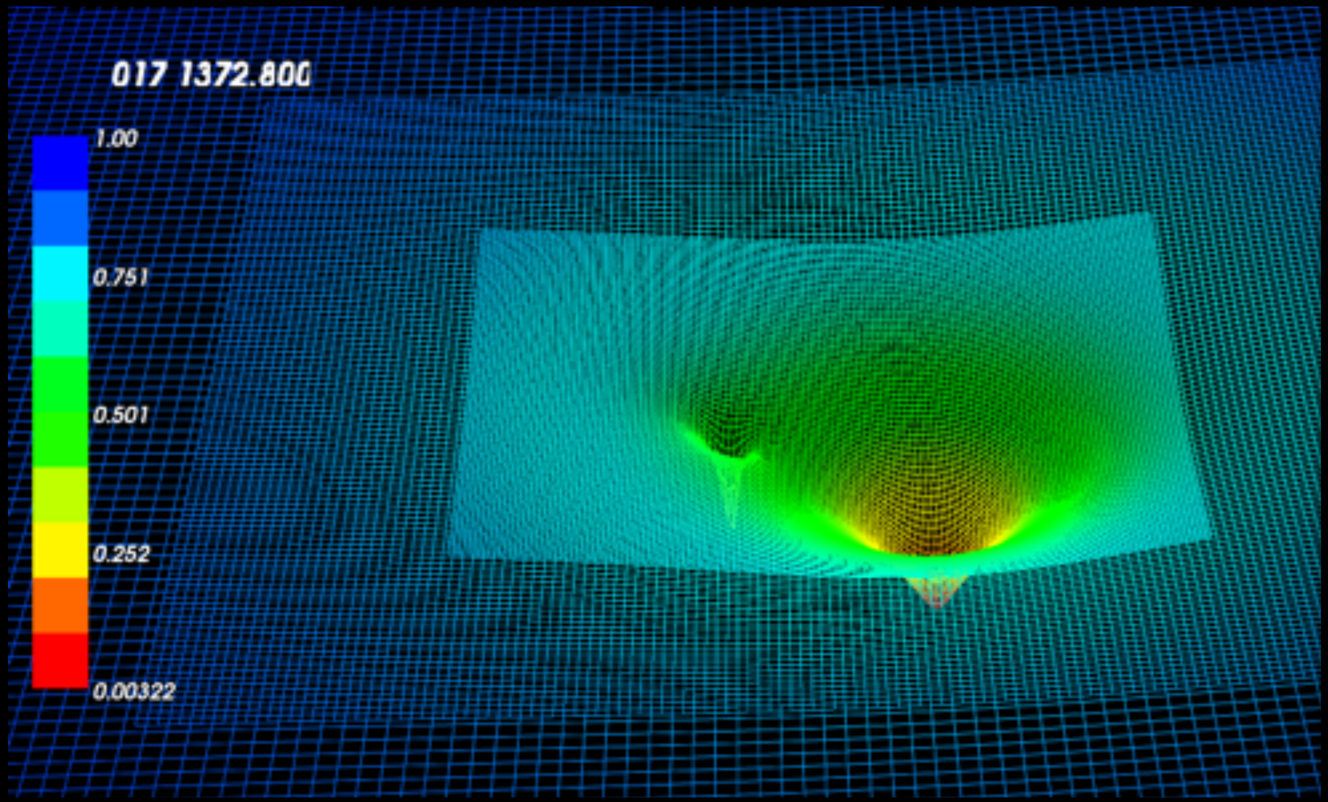
What about the collision?

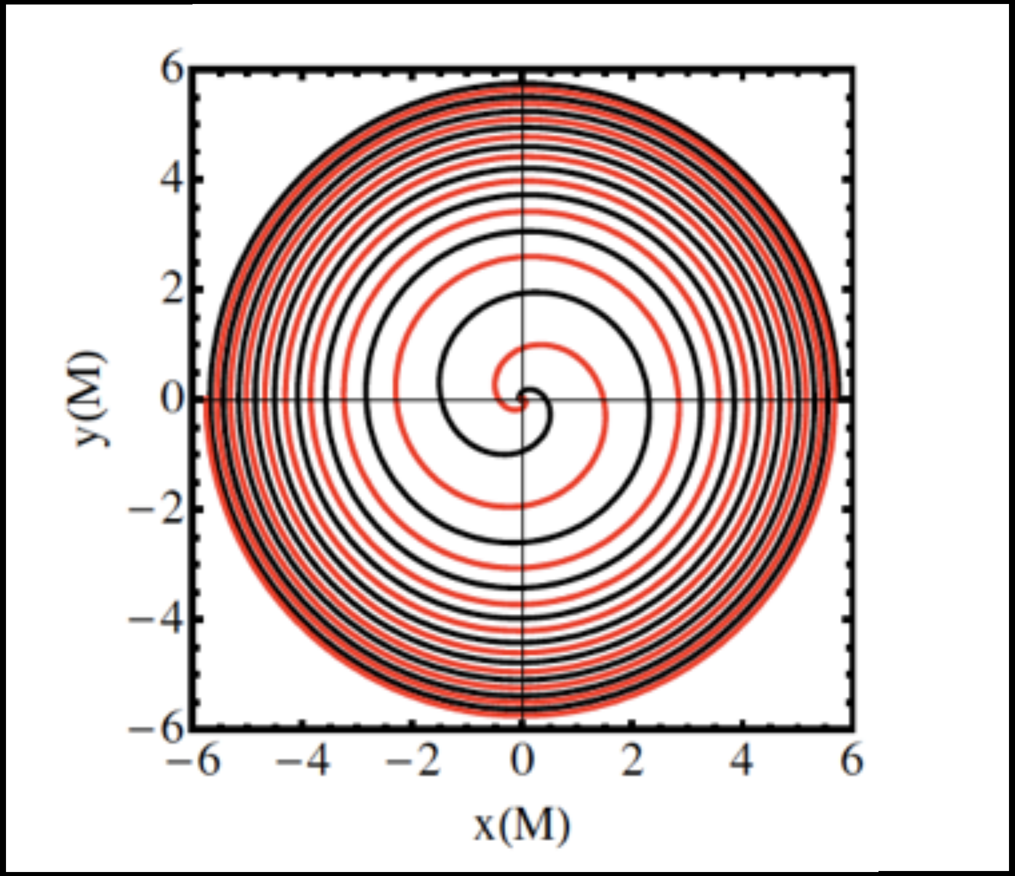
Need to solve full Einstein equations for two BHs…

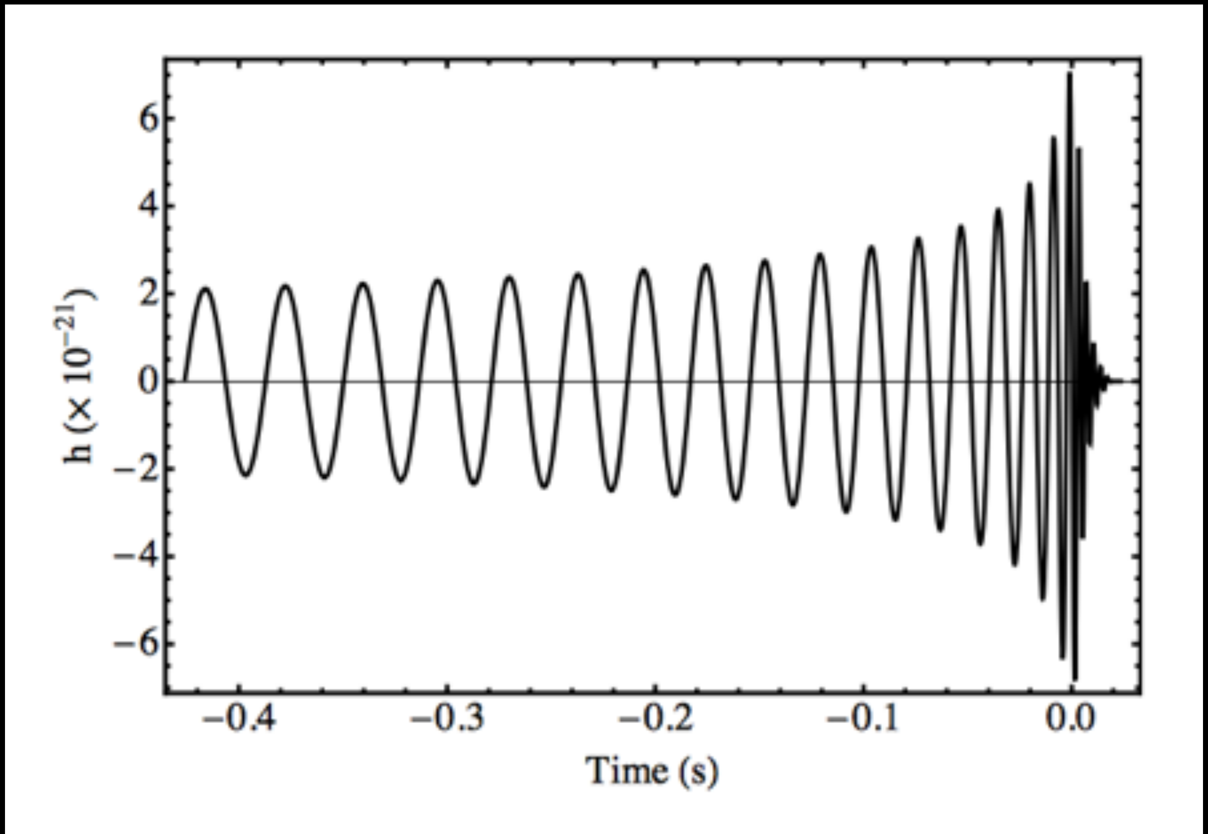
Numerical relativity

numerically solve
full Einstein equations
on 3D (mesh-refined) grids

Requires weeks to months
on 100s of cores.





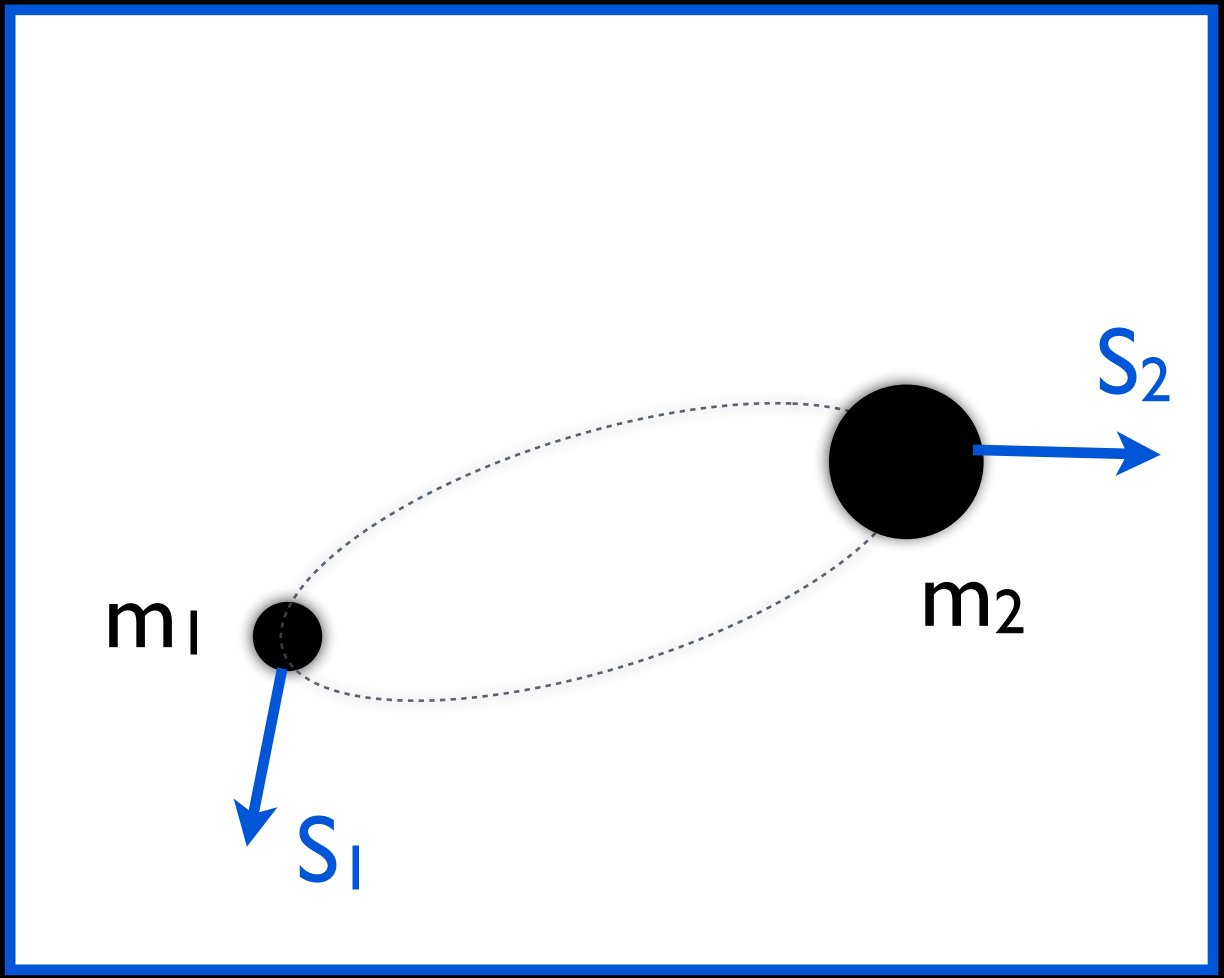


Masses: m_1, m_2
Spins: $\mathbf{S}_1, \mathbf{S}_2$

(8 parameters)

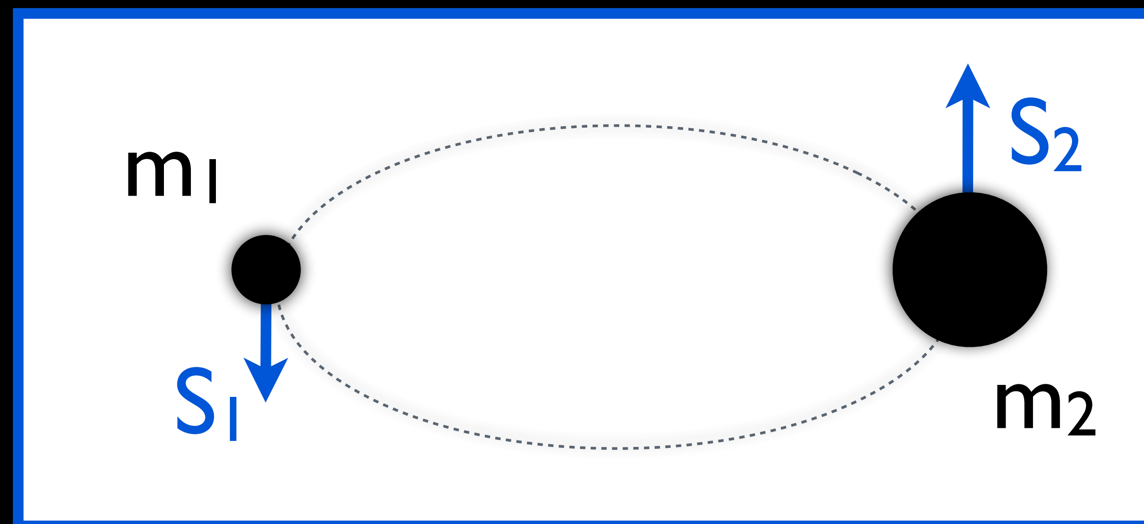
useful combinations:

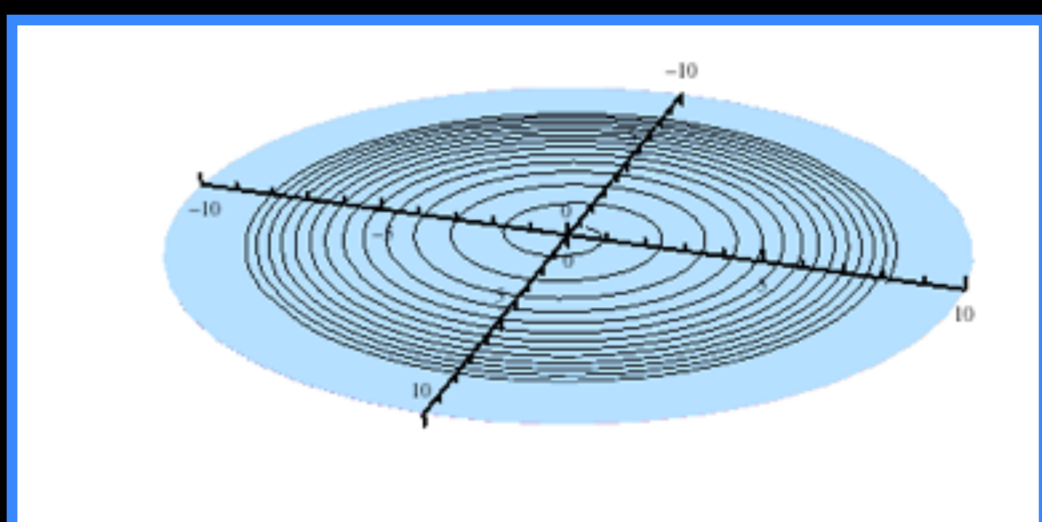
$$M = m_1 + m_2$$
$$q = m_2 / m_1$$
$$\eta = m_1 m_2 / M^2$$
$$\chi = S/m^2$$



Plus: distance, sky location, orientation, polarisation

Aligned spins





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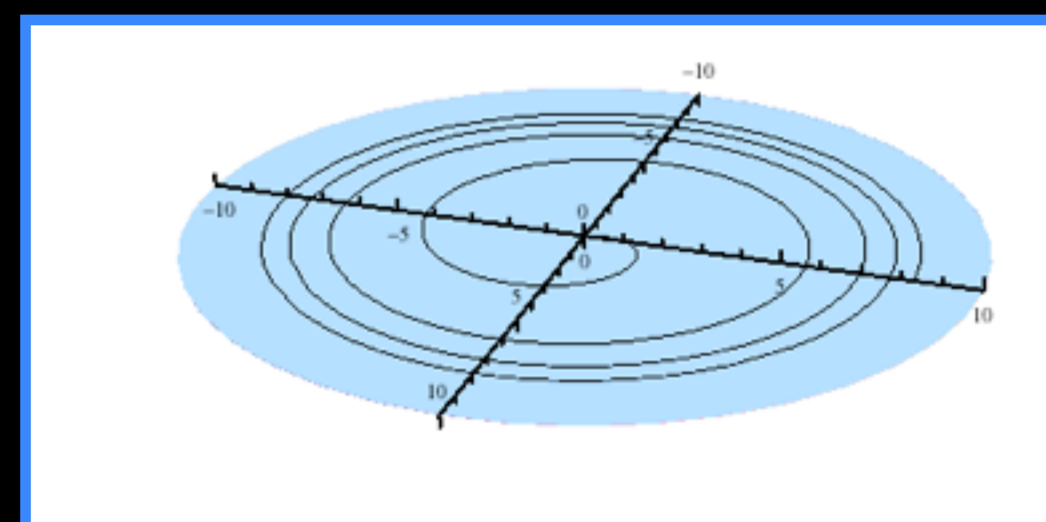
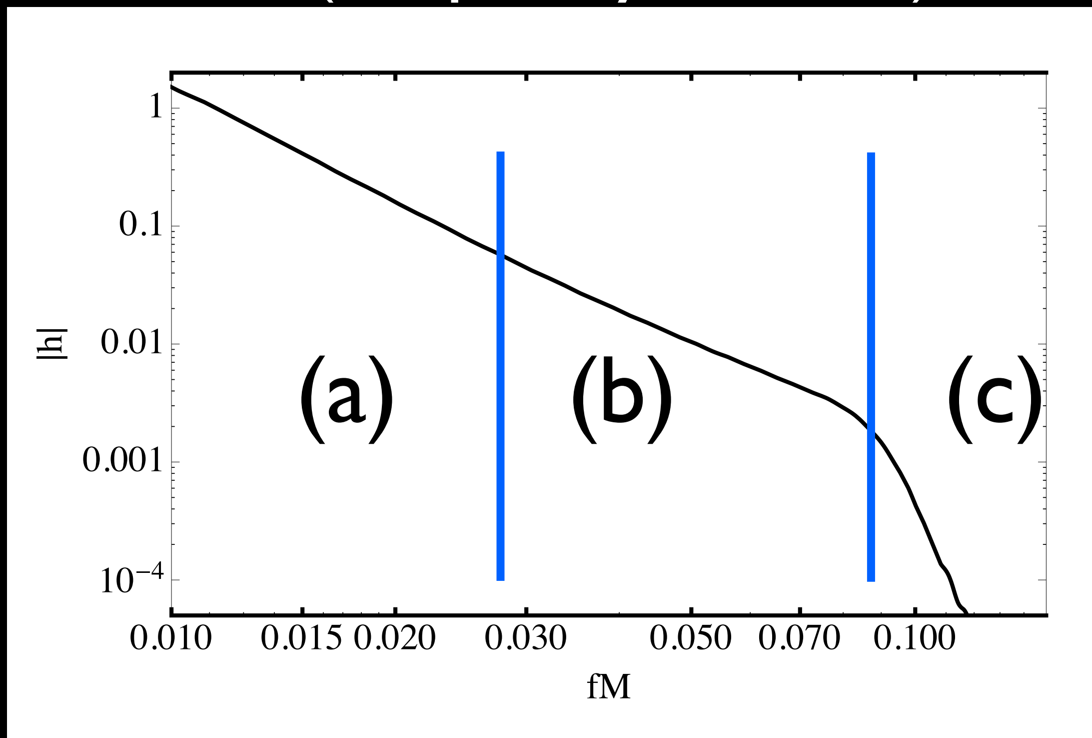


↓↓

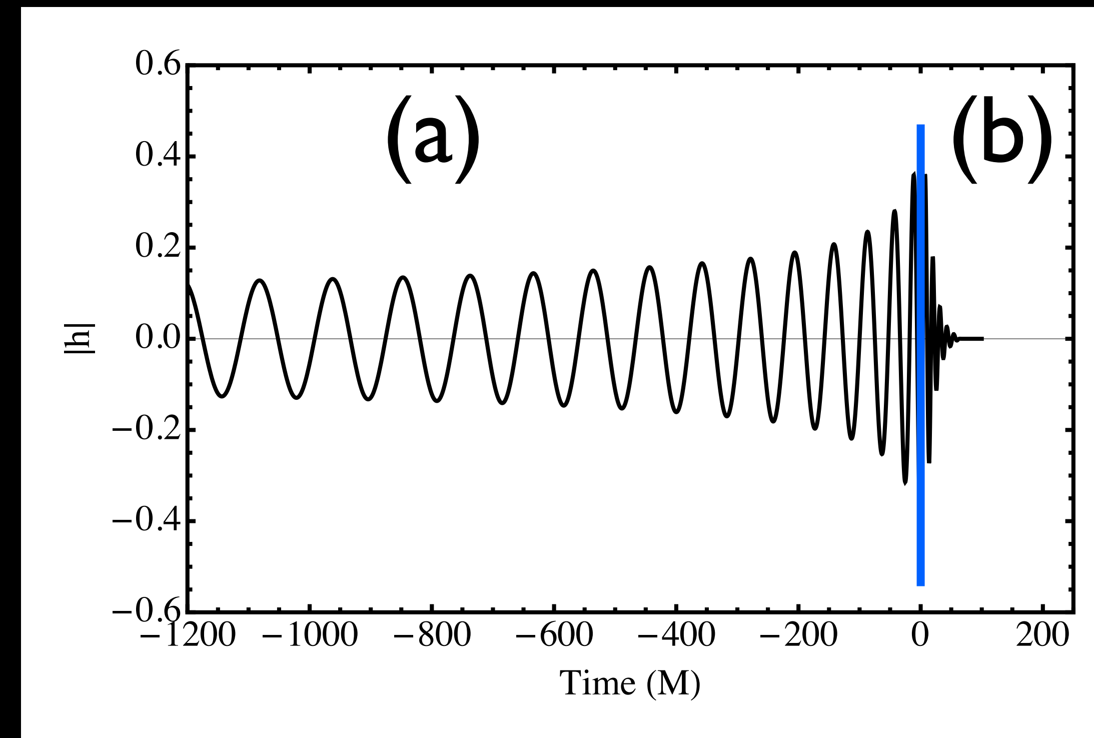
(Dominant spin effect is a weighted sum of the spins)

IMRPhenom
(frequency domain)

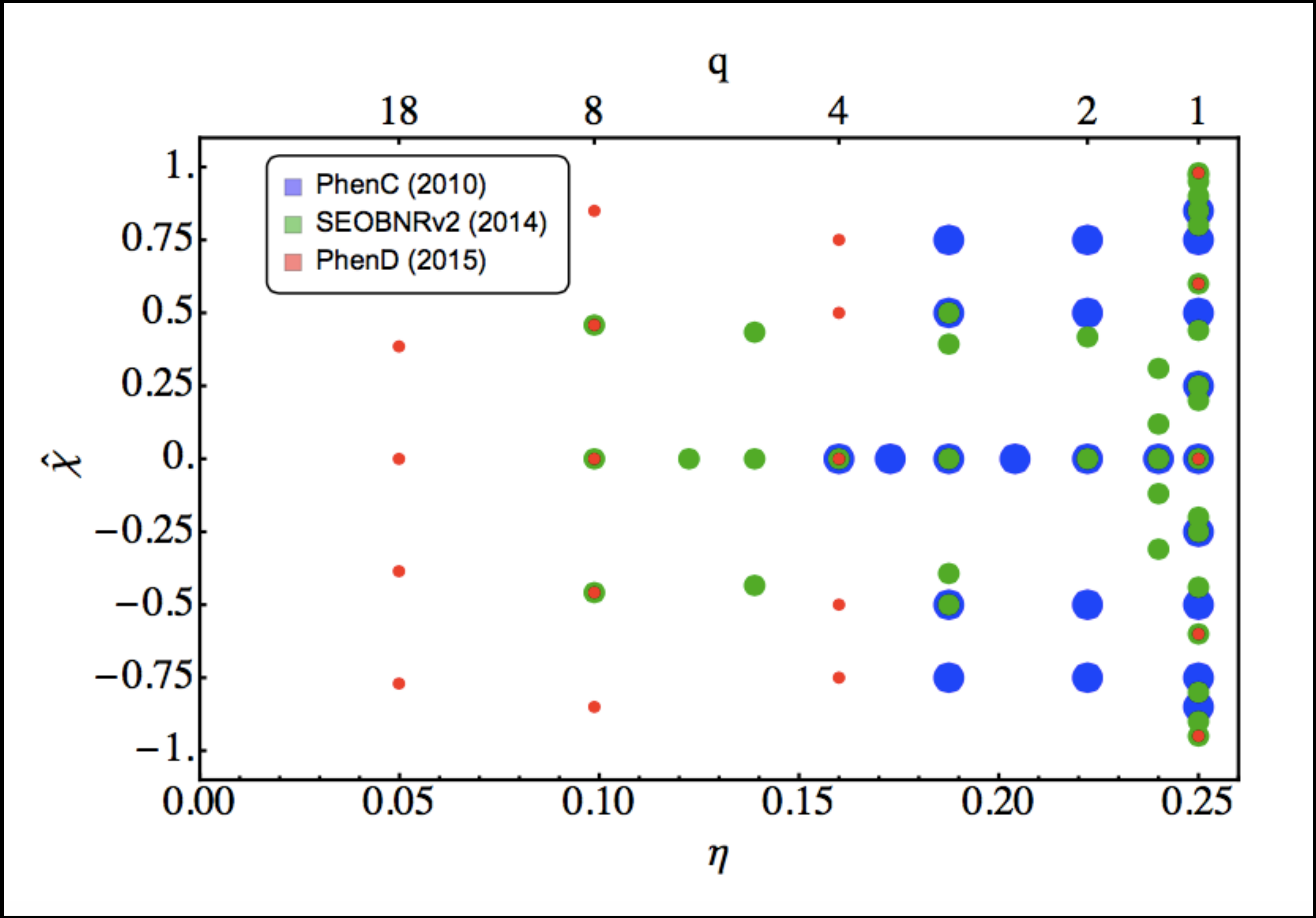


- (a) PN-based ansatz
- (b) phenomenological fit (based on NR behaviour)
- (c) FFT of ringdown waveform (Lorentzian)
- Analytic: fast

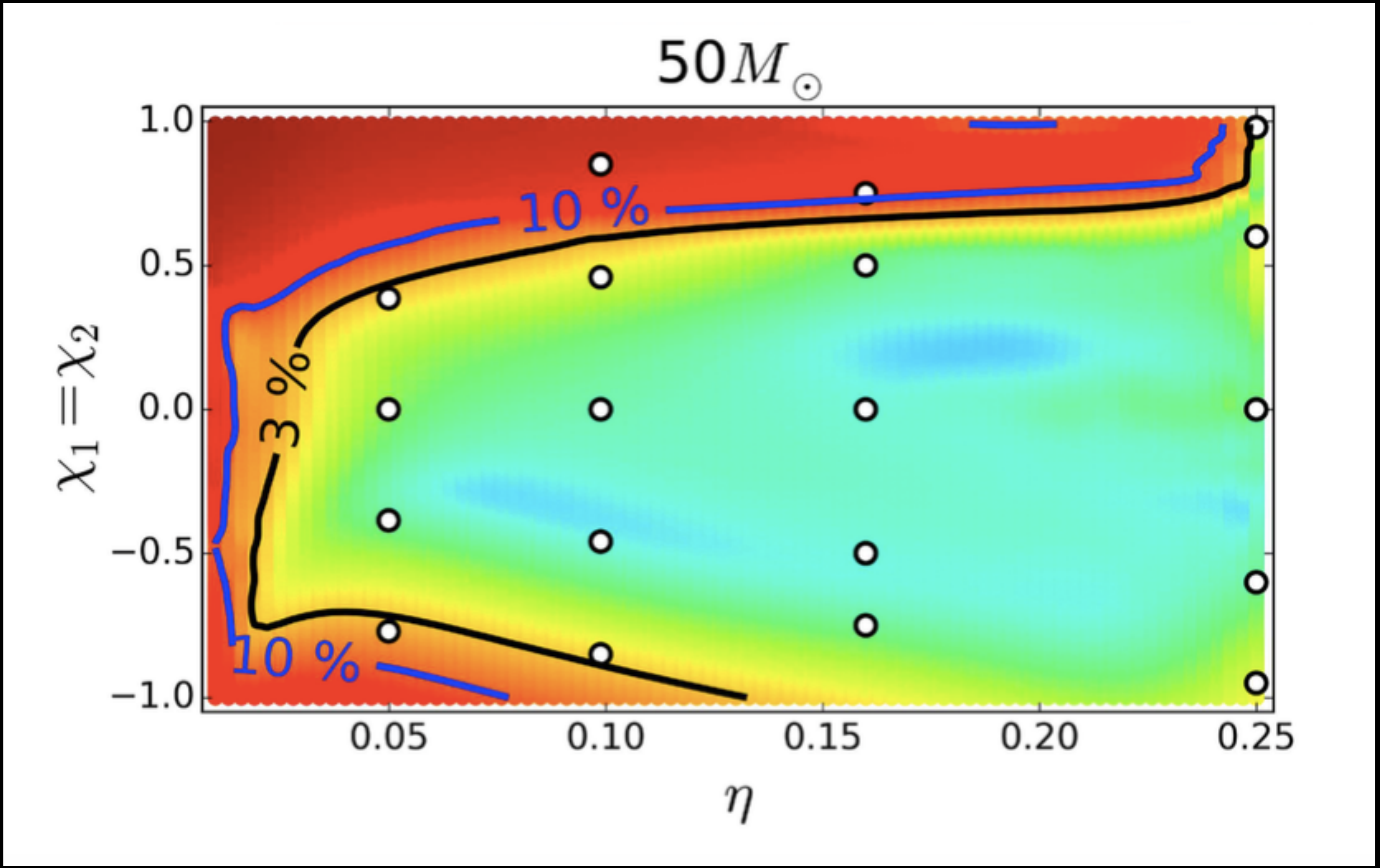
EOBNR
(time domain)



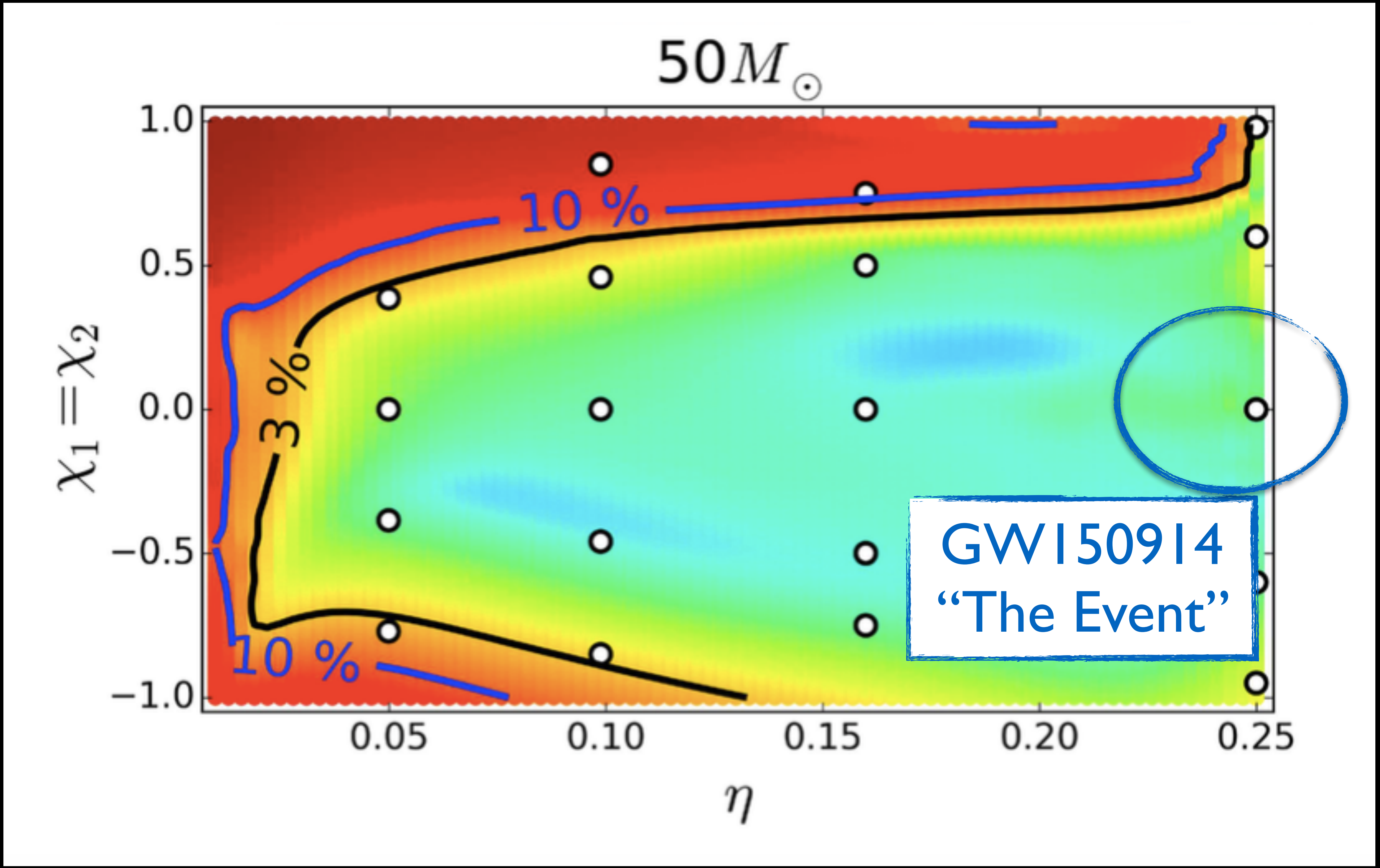
- (a) EOB + terms tuned to NR waveforms
- (b) Smooth transition to ringdown
- Includes both spins
- Numerically solve ODEs: slow
- Speed-up: Reduced-order models

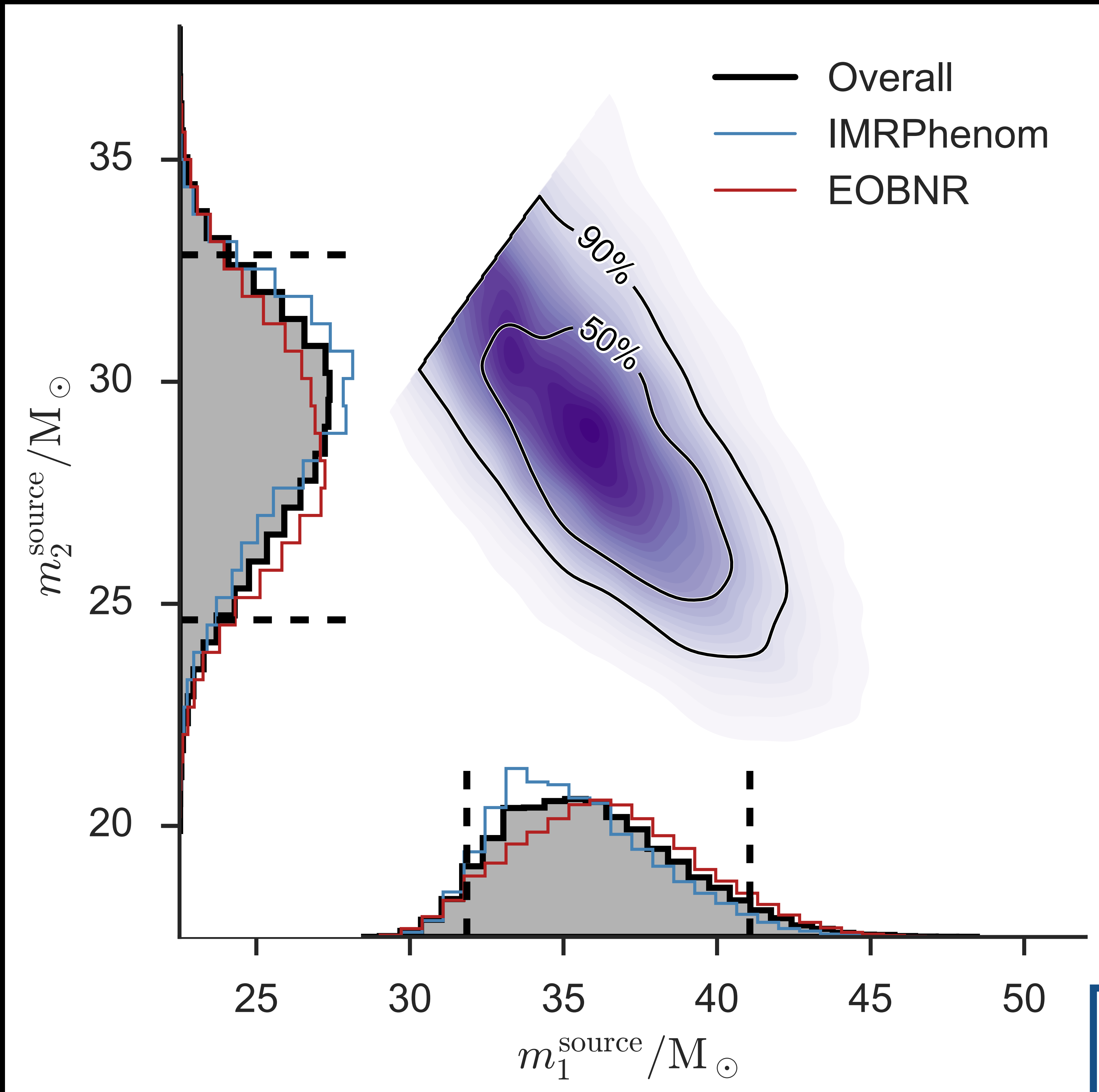


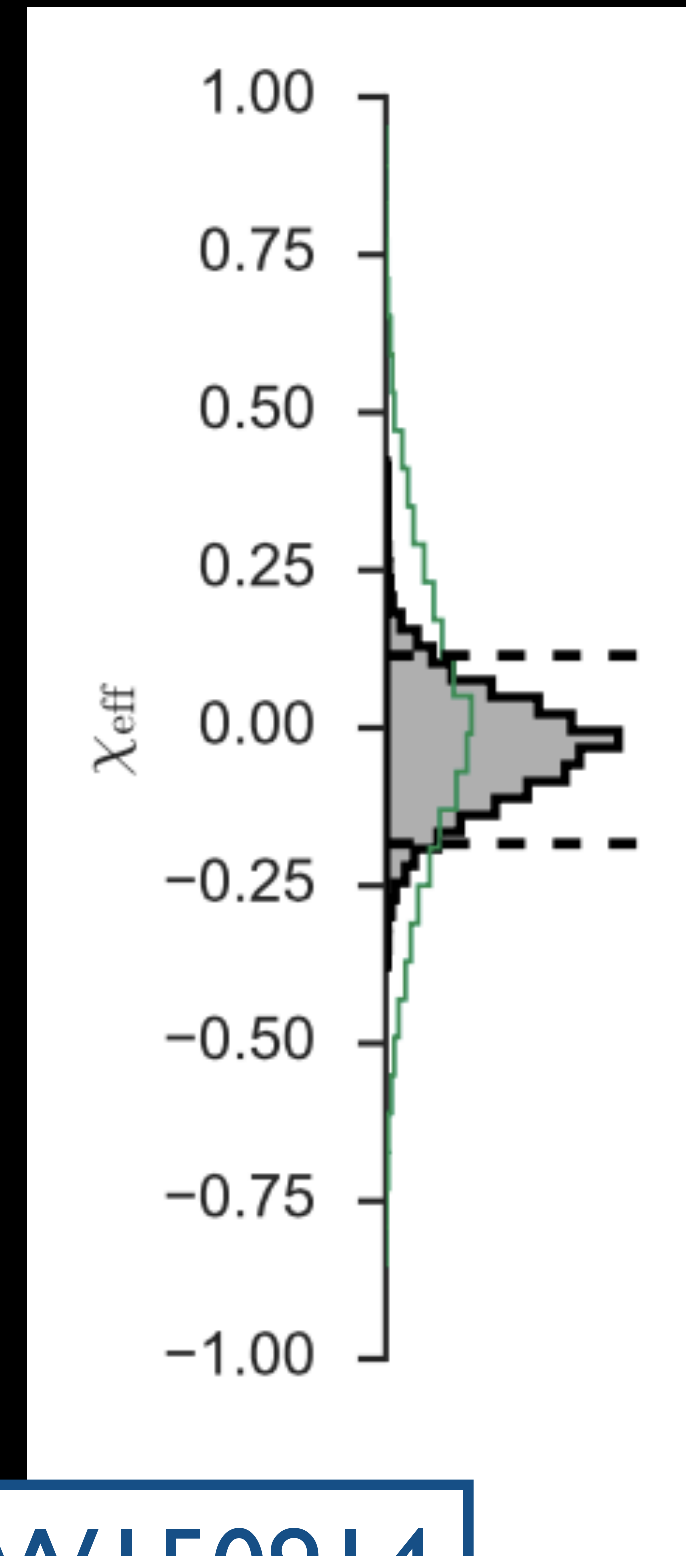
[Khan, *et. al*, PRD **93**, 044007 (2016)]



[Khan, *et. al*, PRD **93**, 044007 (2016)]


$50M_\odot$
1.0
0.5
0.0
−0.5
−1.0
$\chi_1 = \chi_2$
0.05
0.10
0.15
0.20
0.25
η
10 %
3 %
10 %
GW150914
"The Event"

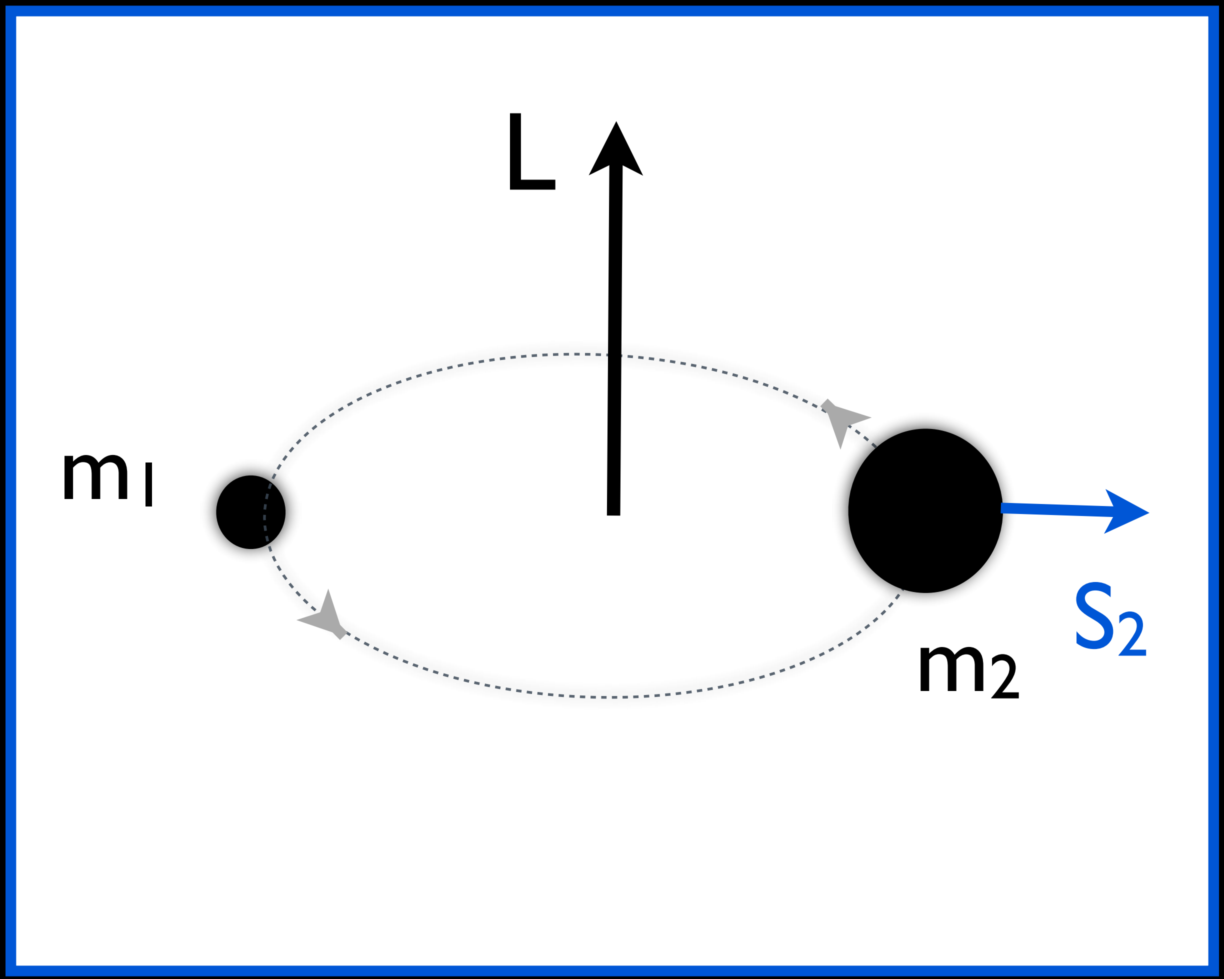


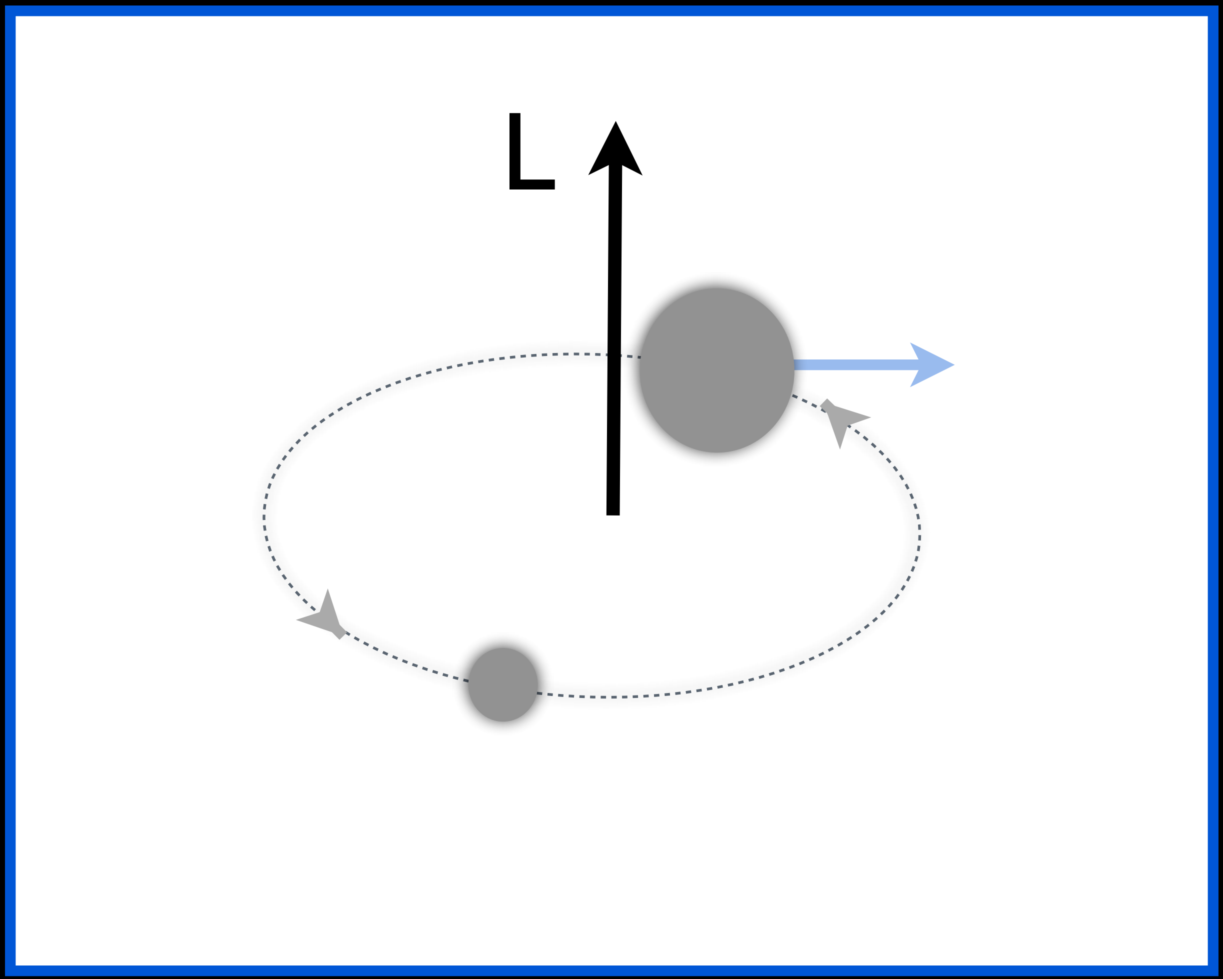


[LVC, PRL 116, 241102 (2016)]

GW150914
"The Event"

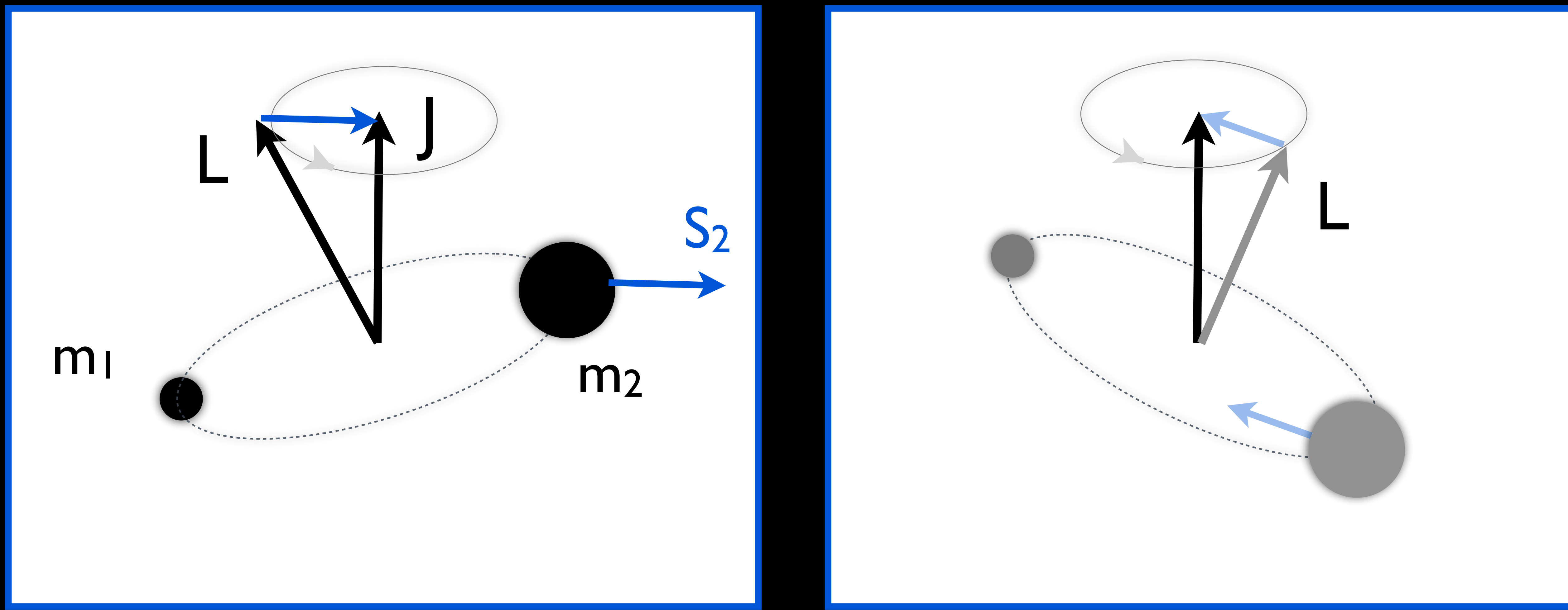
Orbital precession





Newtonian gravity:
L, S_1, S_2 remain fixed

Orbital precession



General relativity

(L, S_1, S_2) precess around J

Orientation dependence

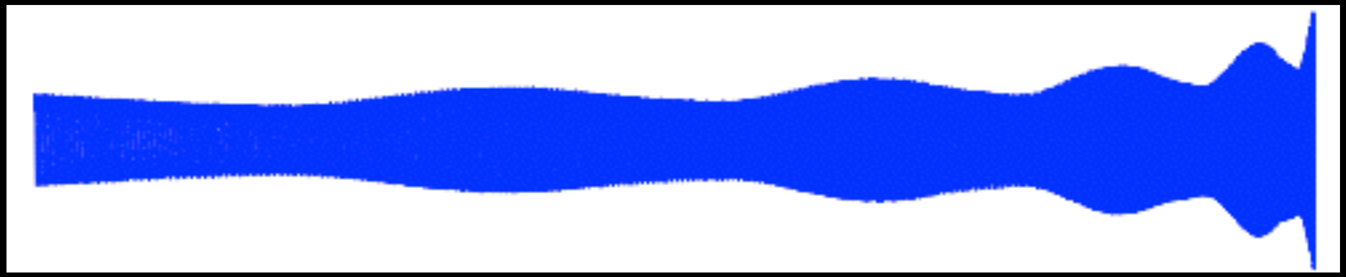
q=3, $|\mathbf{S}_2|$ = 0.75 (in plane)

Observer aligned with J

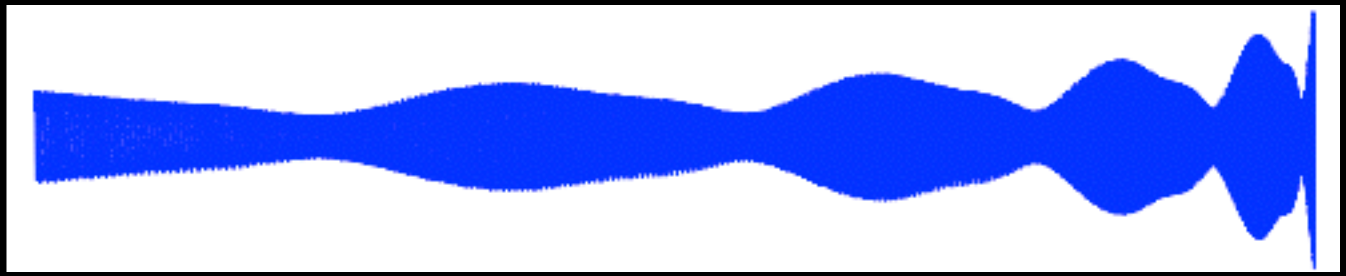
Orientation dependence

q=3, $|S_2|$ = 0.75 (in plane)

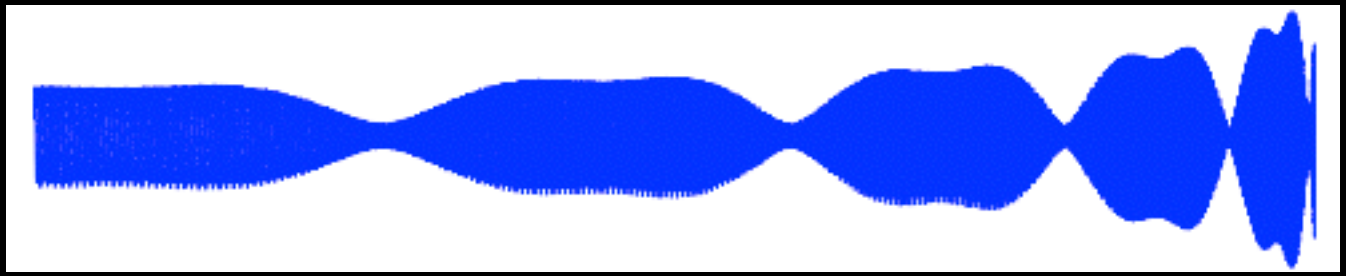
Observer aligned with J



Observer inclined $\pi/6$ to J

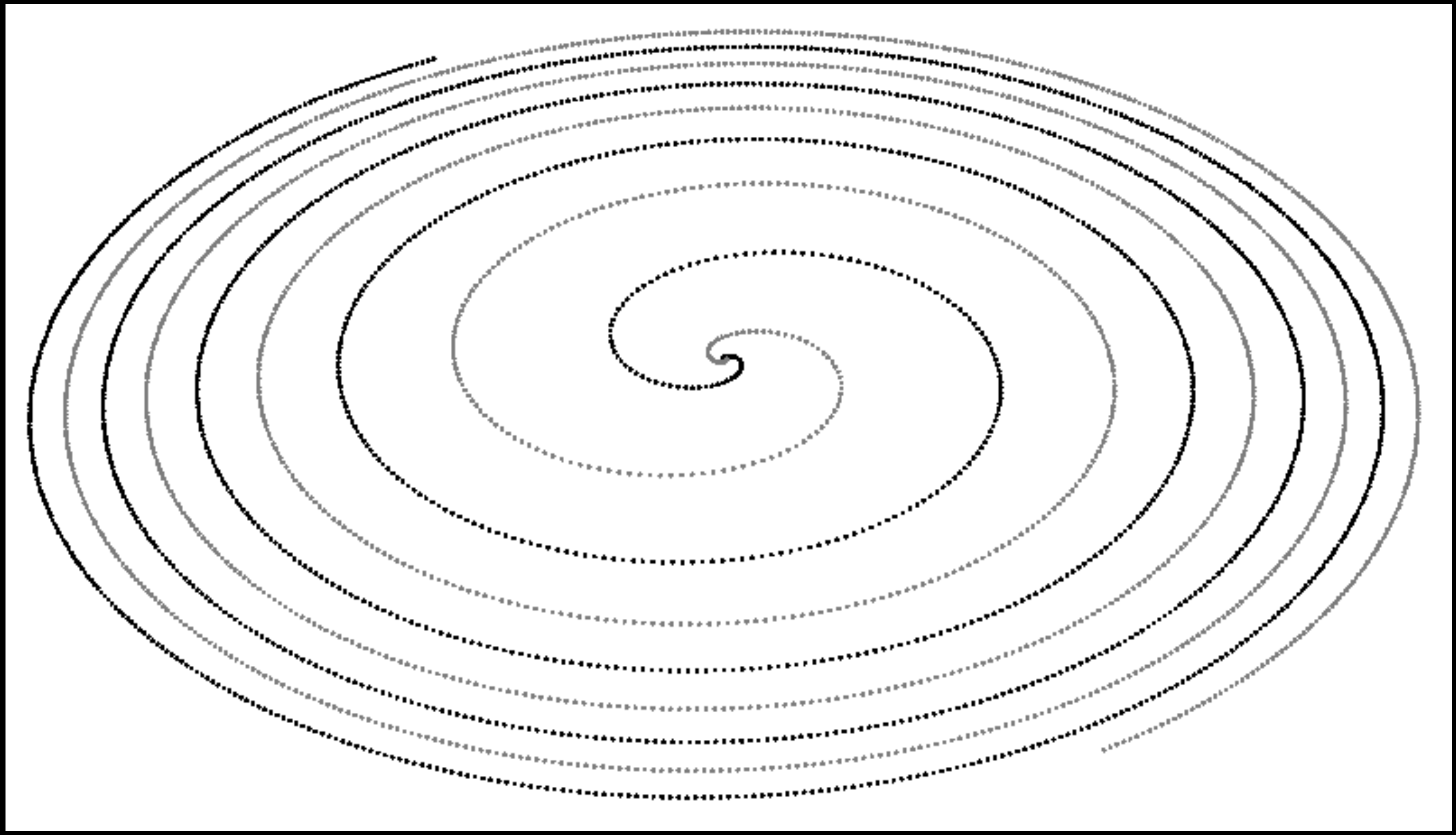


Observer inclined $\pi/3$ to J

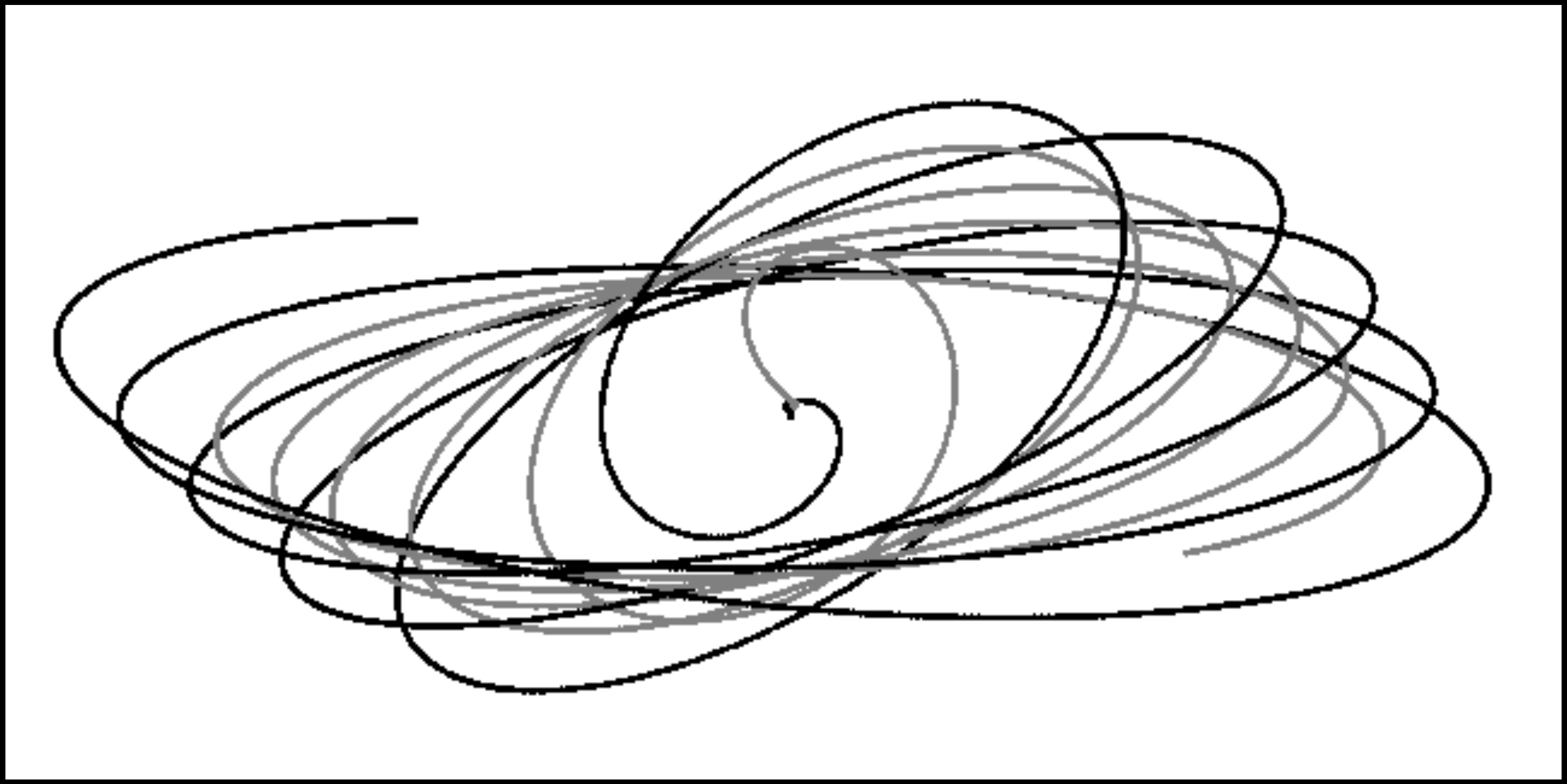


Observer inclined $\pi/2$ to J

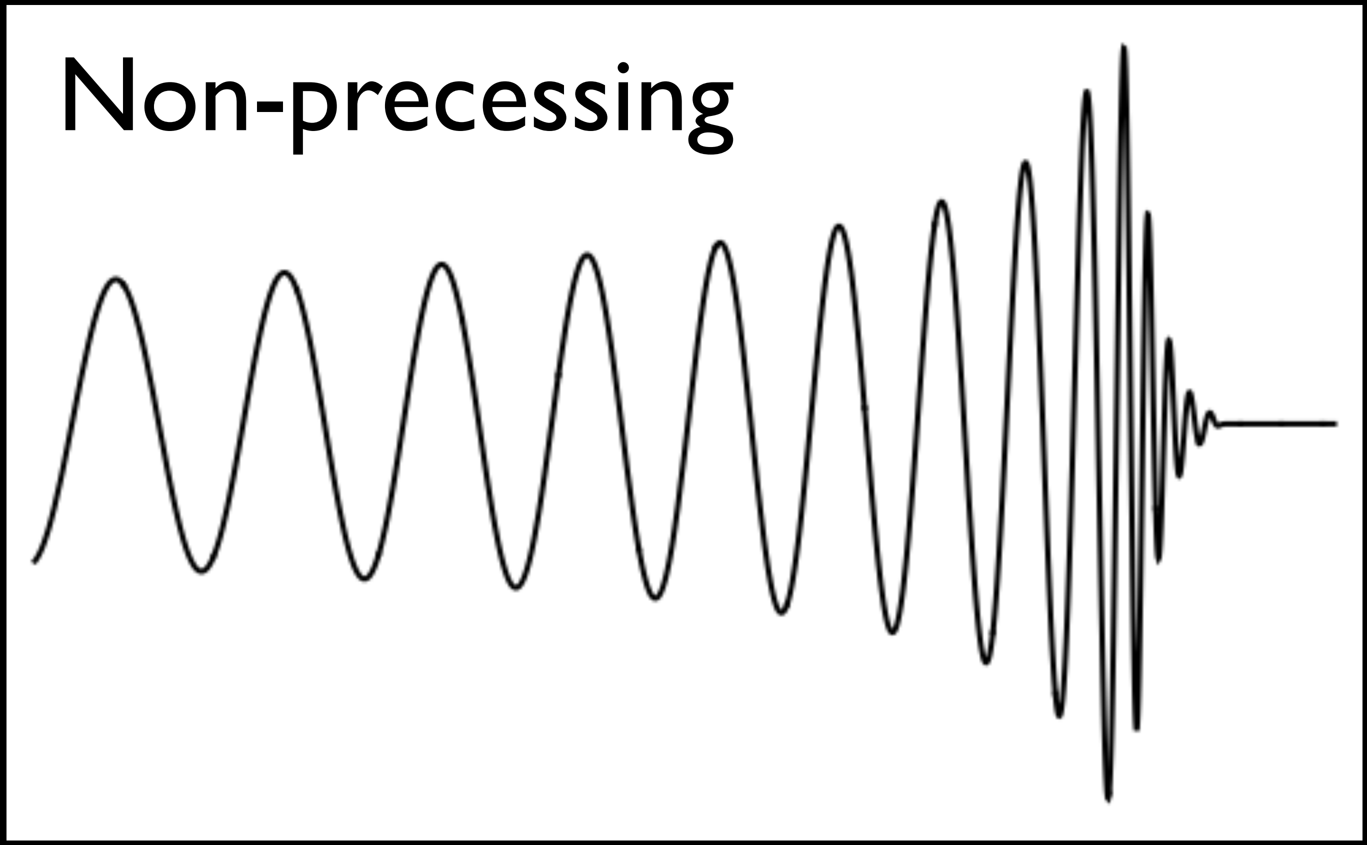
Equal-mass nonspinning BBH consistent with GW150914

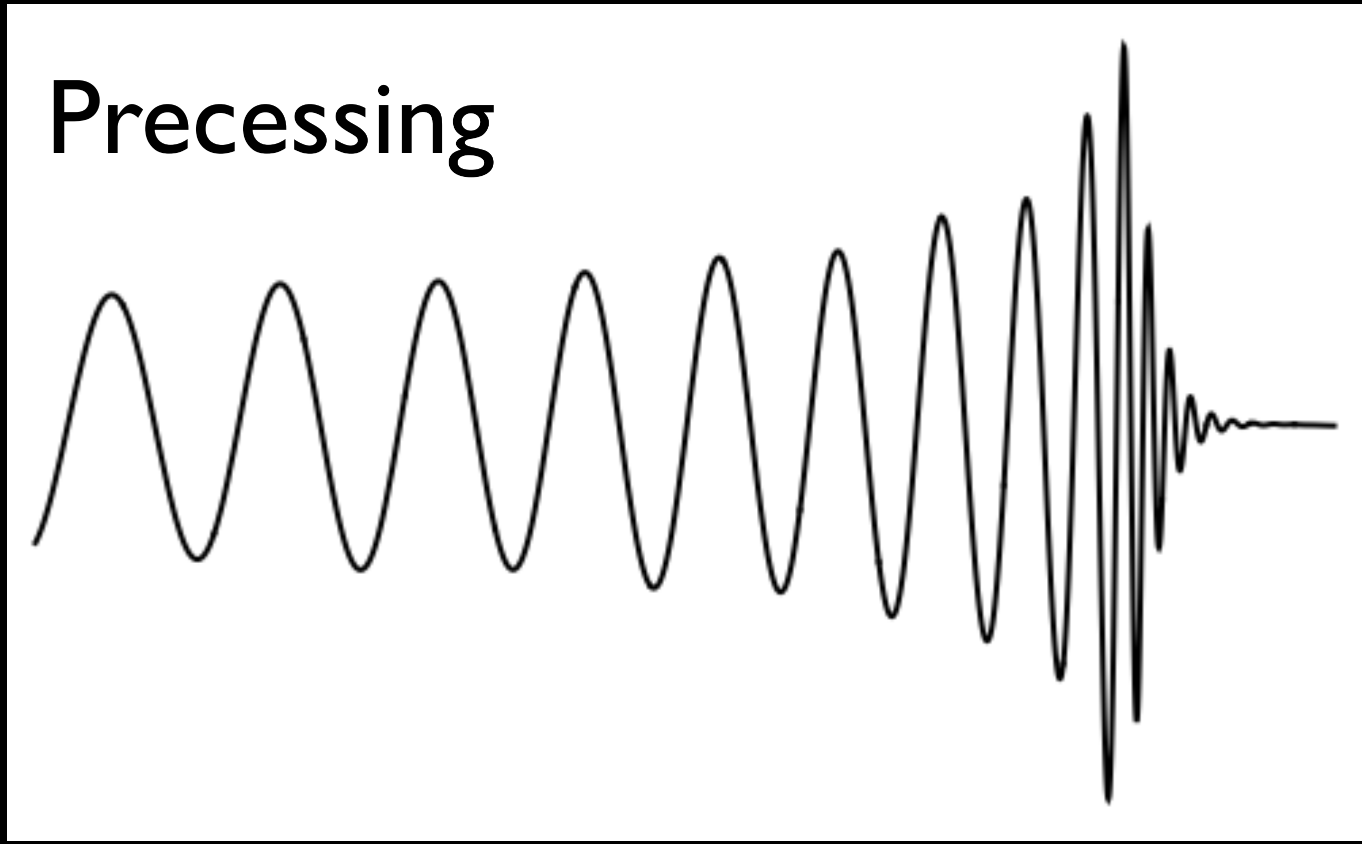


Unequal-mass precessing BBH consistent with GW150914

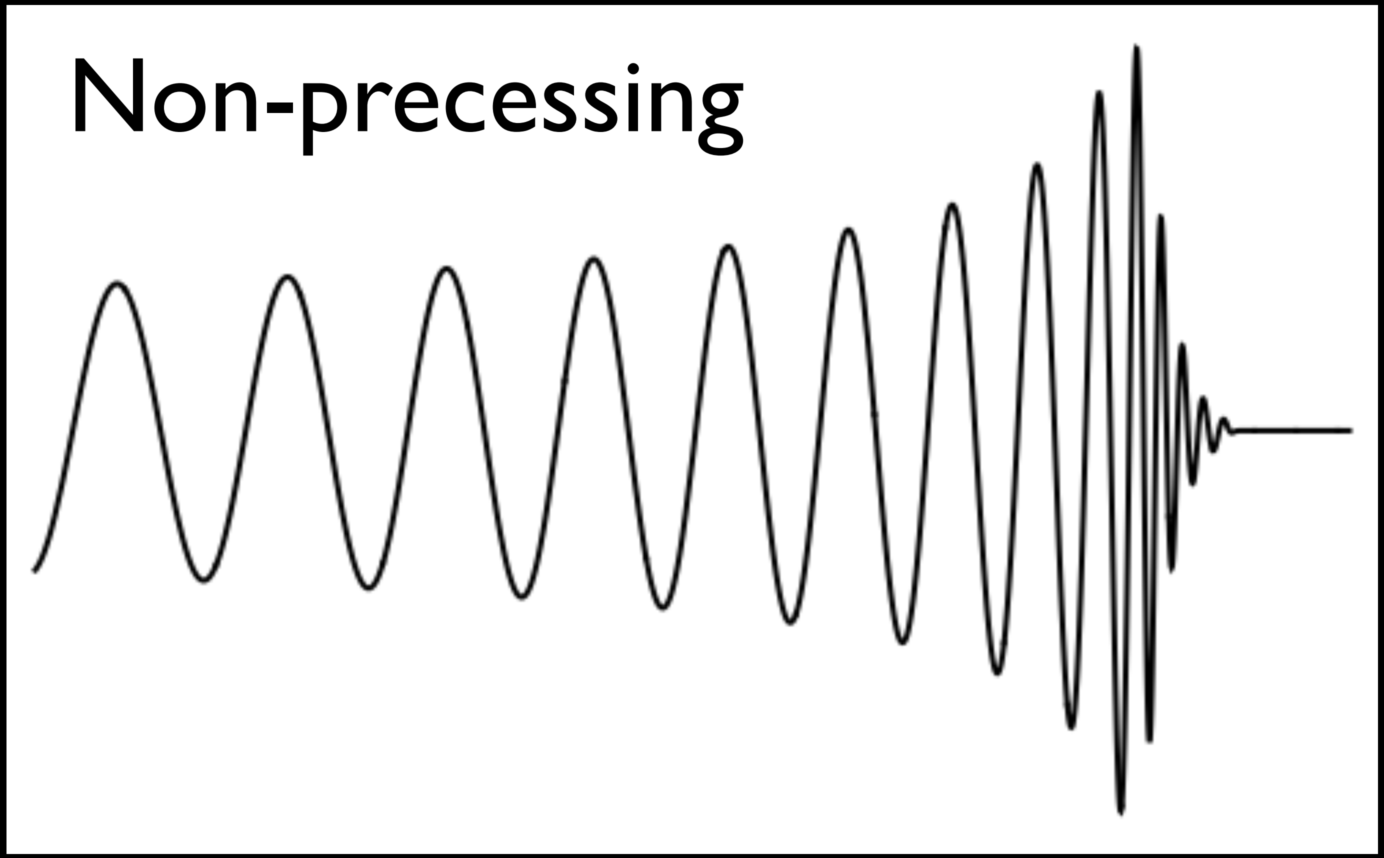


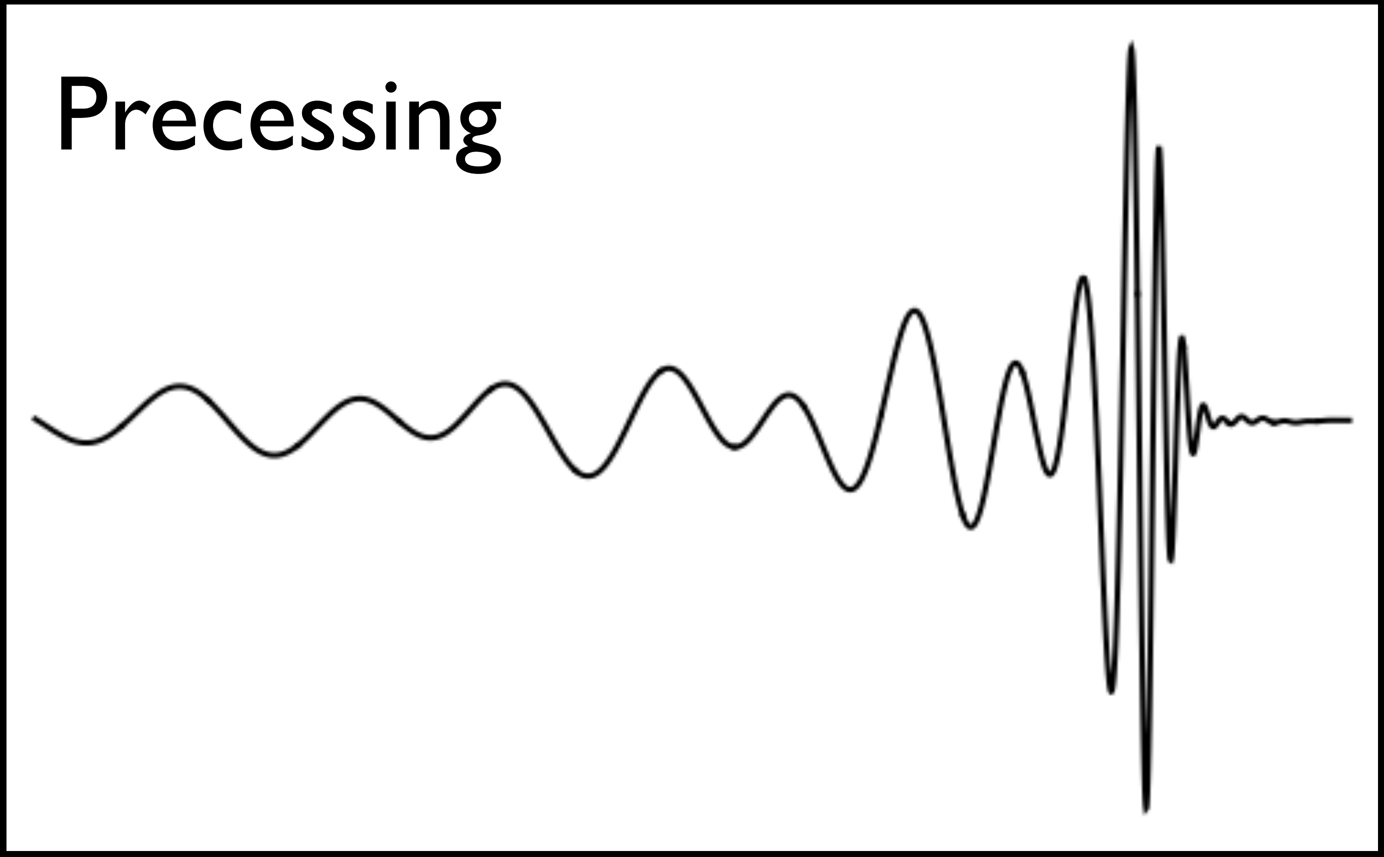
“Face-on” to the source

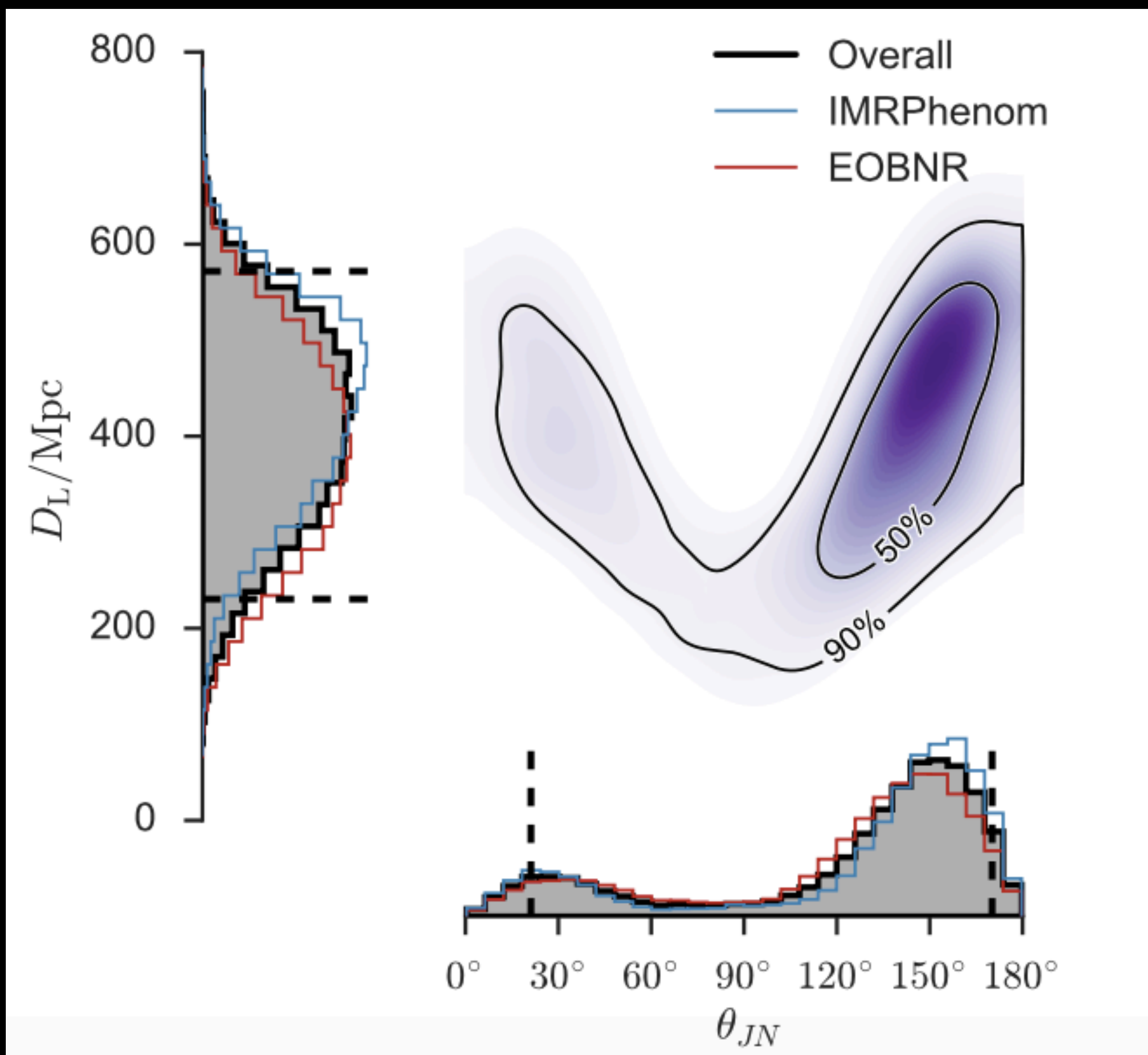


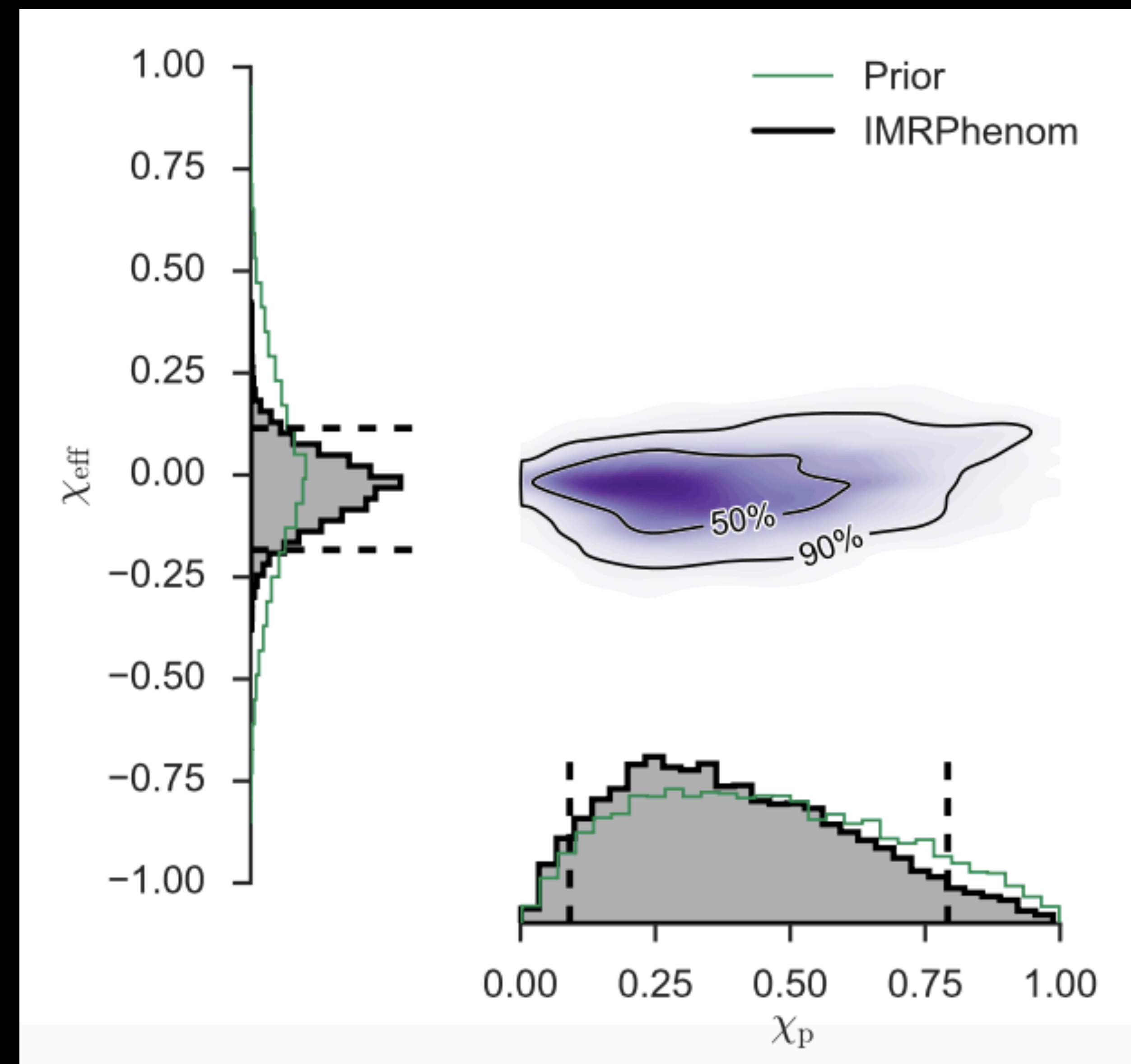


“Edge-on” to the source



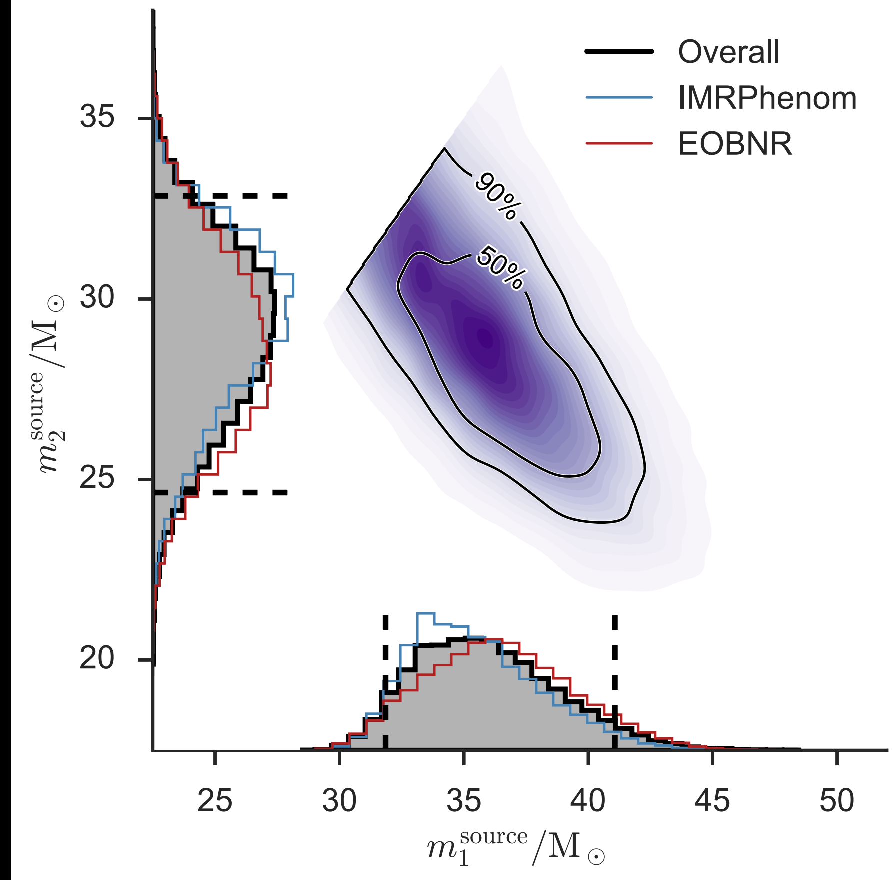




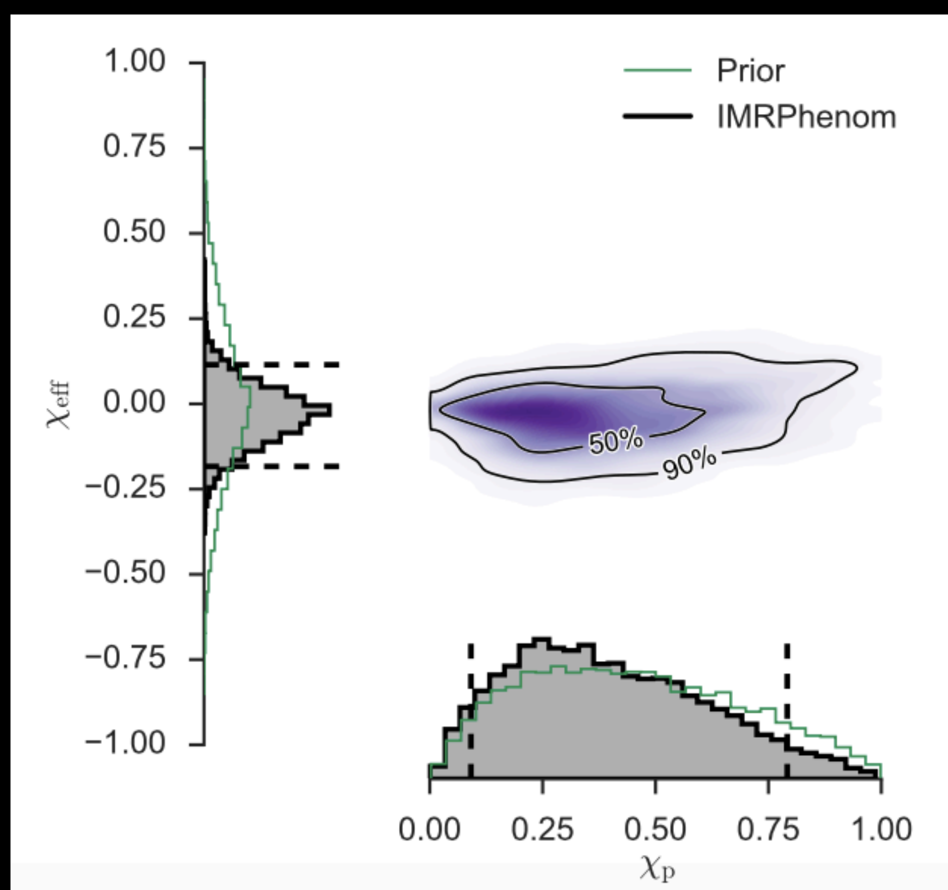


[LVC, PRL **116**, 241102 (2016)]

GW150914

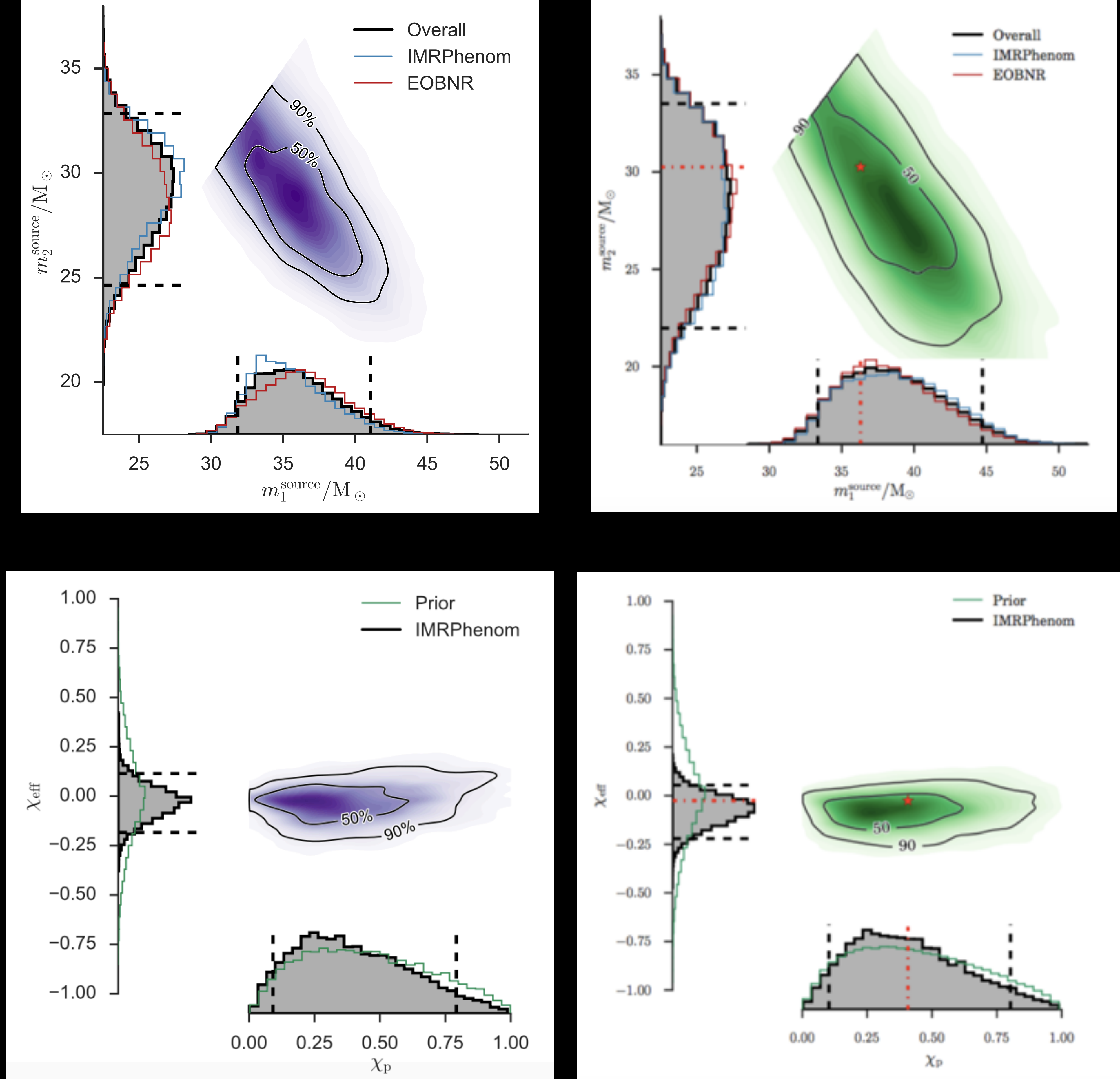


Overall
IMRPhenom
EOBNR
90%
50%
35
30
25
20
$m_2^{\mathrm{source}}/\mathrm{M}_\odot$
25 30 35 40 45 50
$m_1^{\mathrm{source}}/\mathrm{M}_\odot$



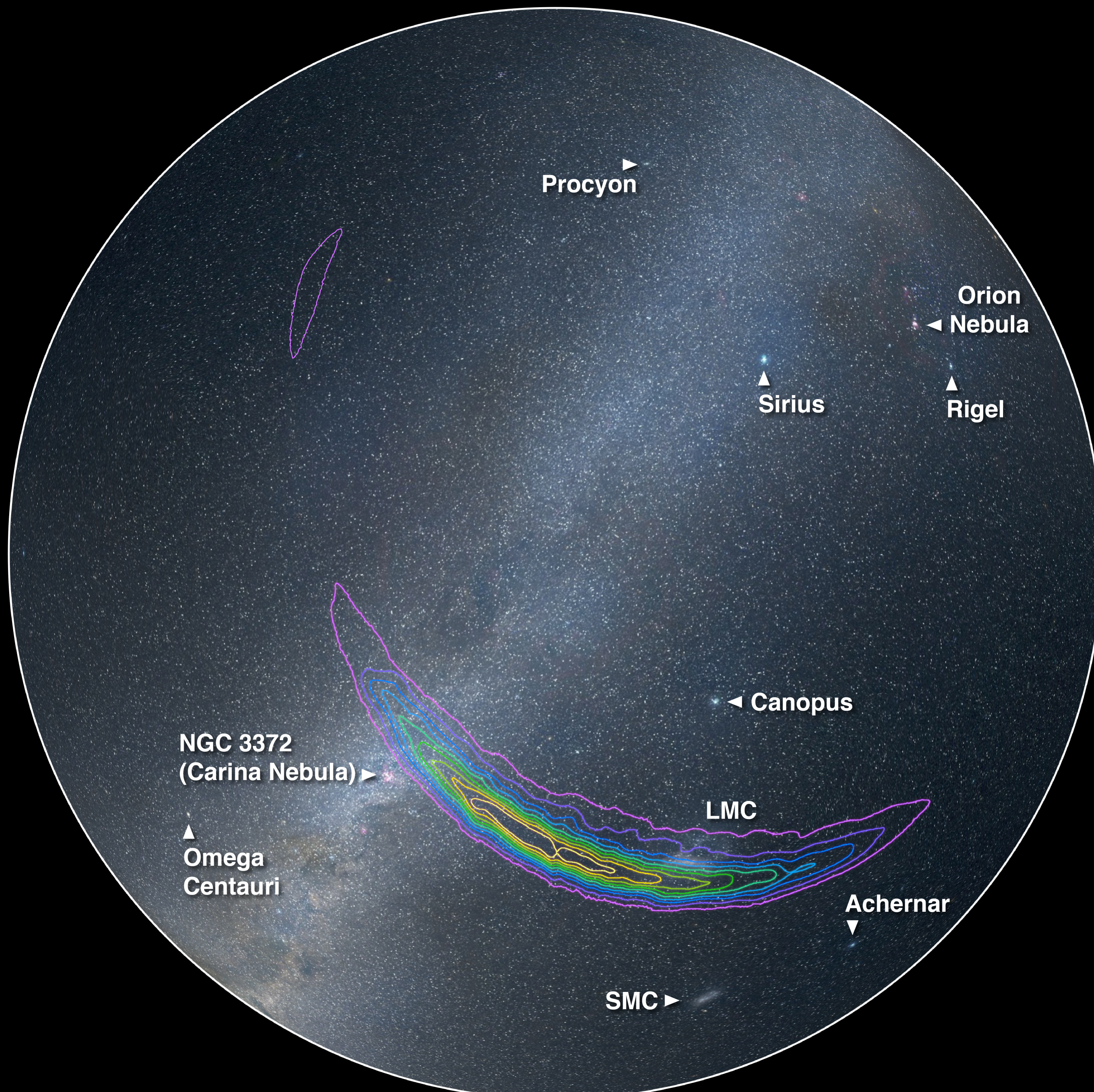
Prior
IMRPhenom
50%
90%
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
χ_{eff}
0.00 0.25 0.50 0.75 1.00
χ_{p}

GW150914

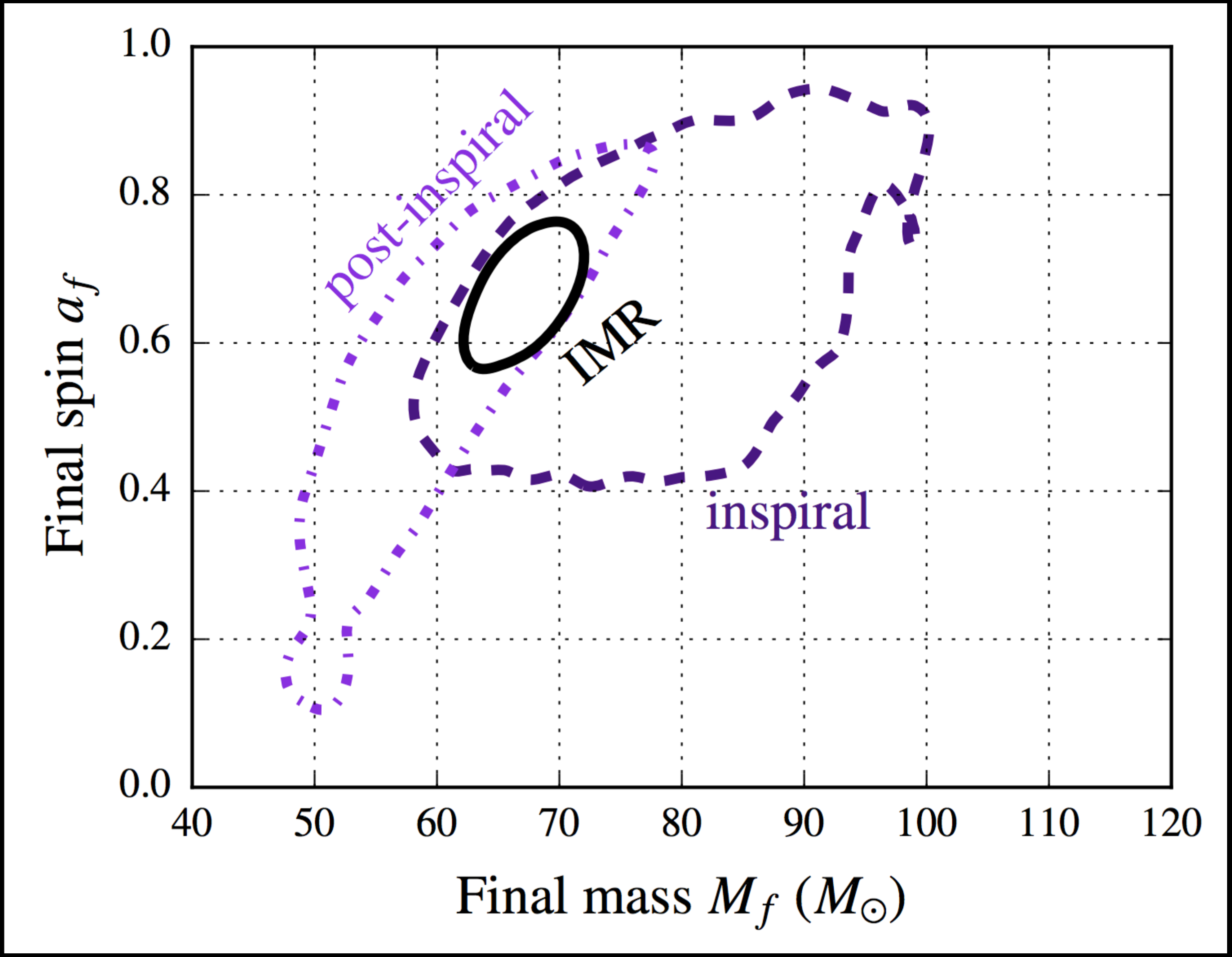


NR Injection

[LVC, in preparation (2016)]

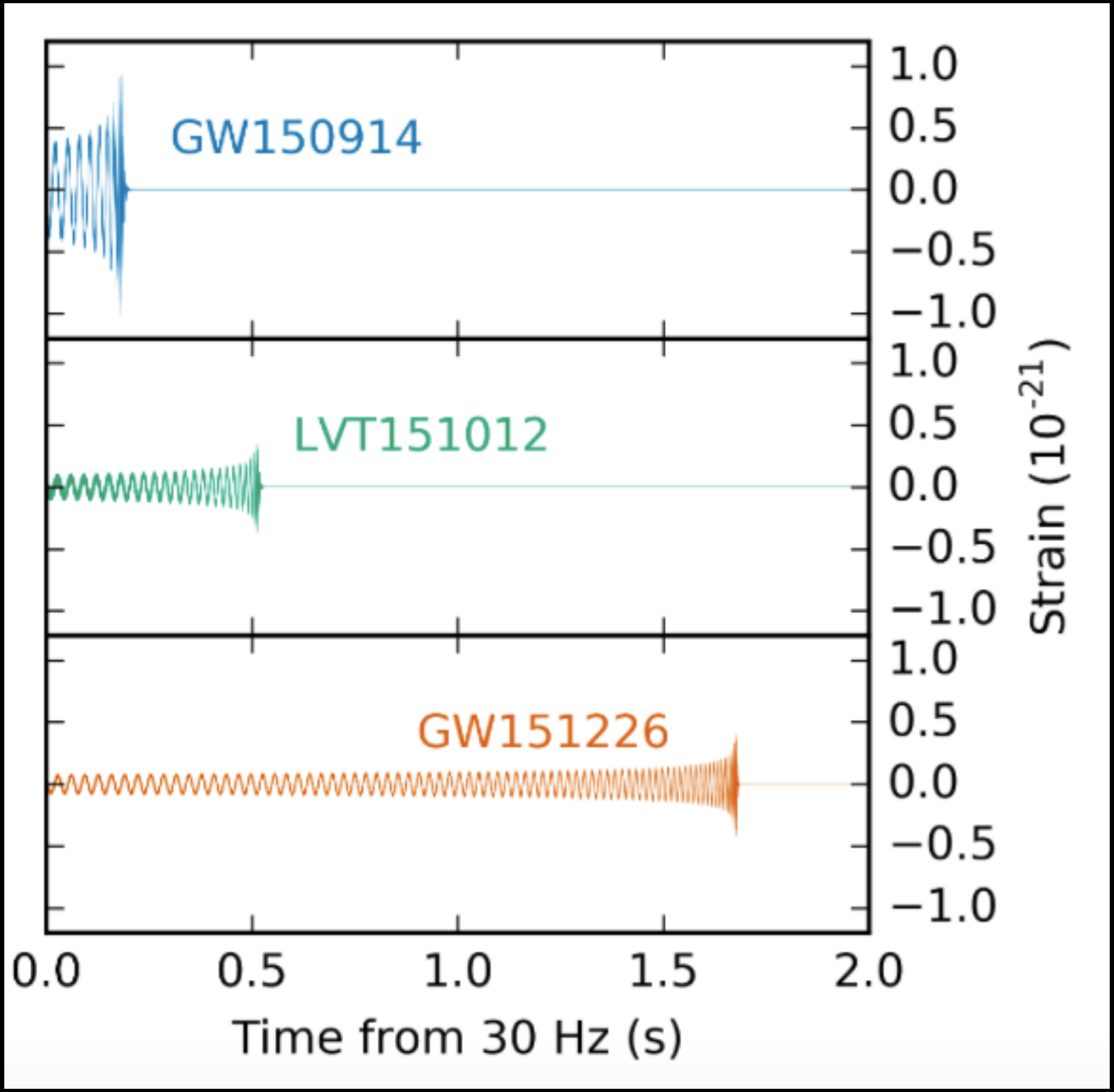


Procyon
Orion
Nebula
Sirius
Rigel
Canopus
NGC 3372
(Carina Nebula)
LMC
Omega
Centauri
Achernar
SMC



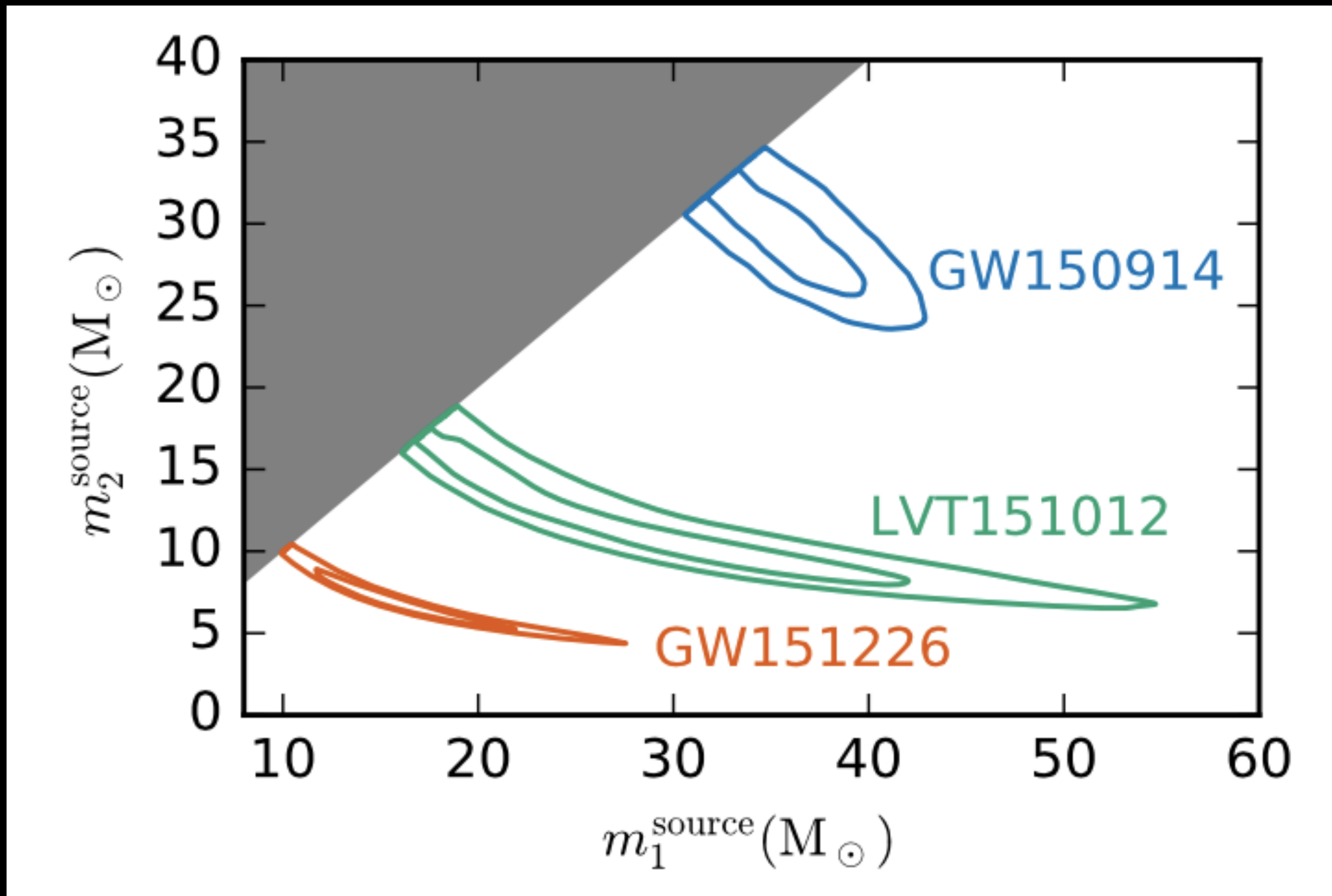
[LVC, PRL **116**, 221101 (2016)]

All observations



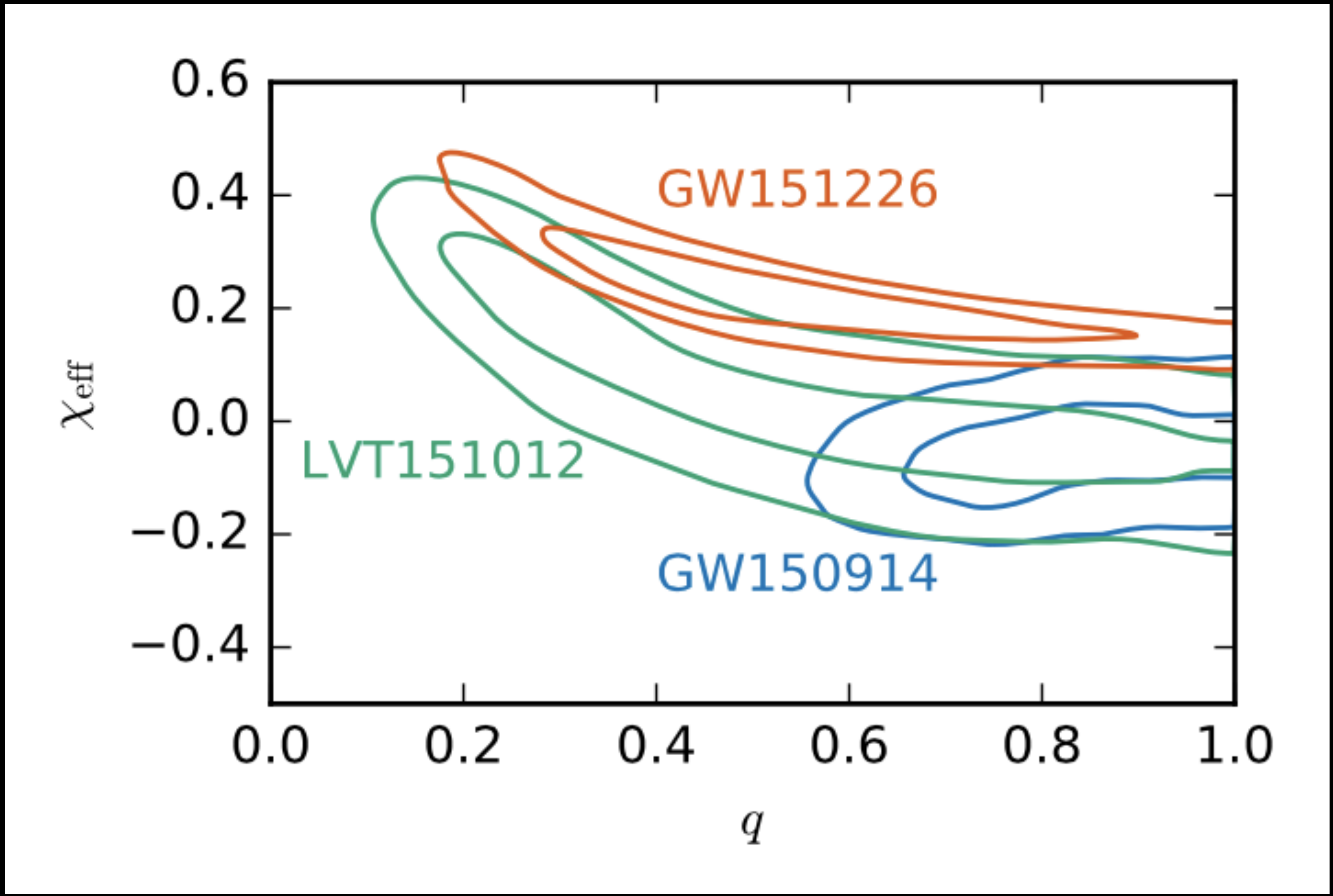
[LVC, arXiv:1606.04856 (2016)]

All observations



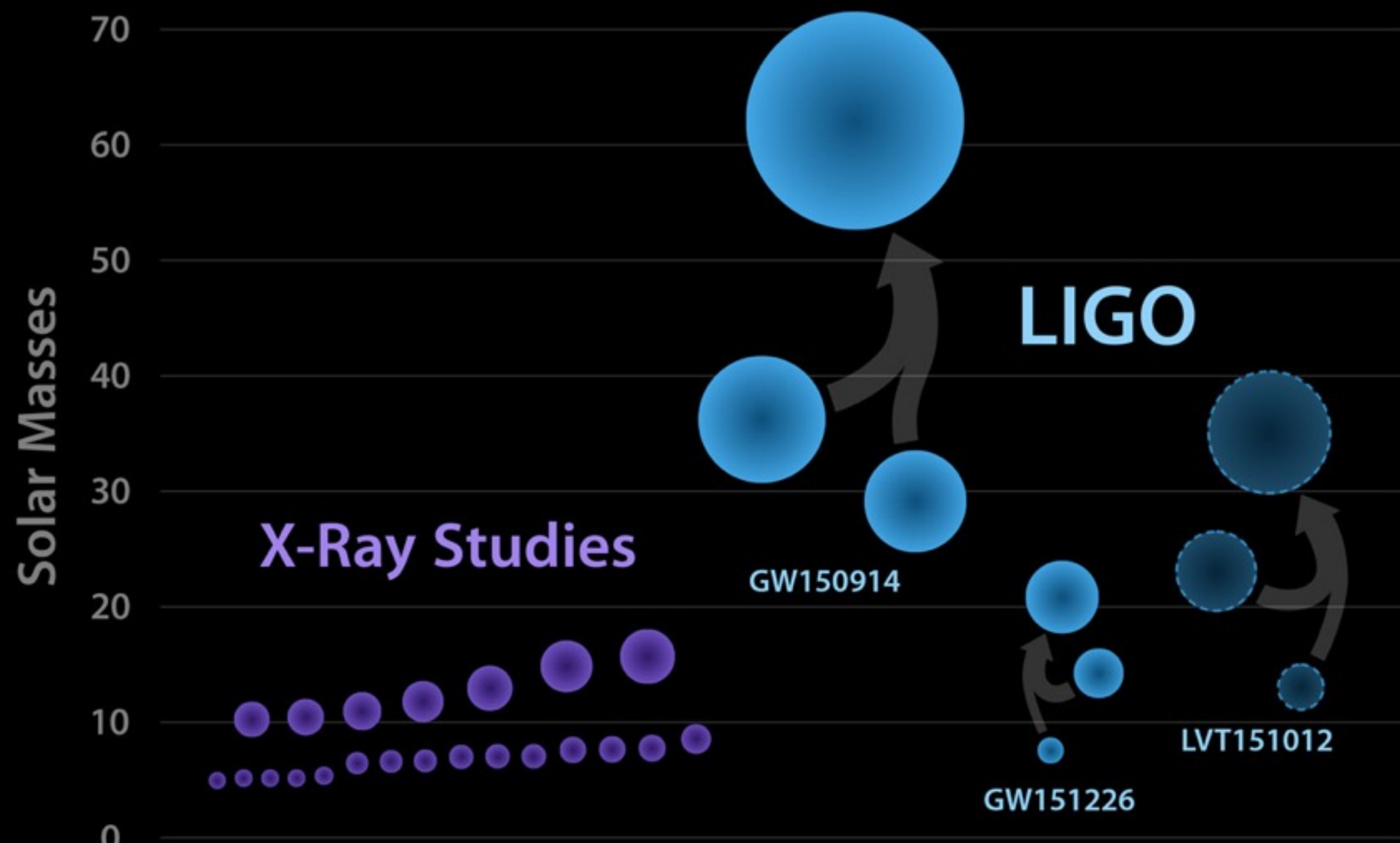
[LVC, arXiv:1606.04856 (2016)]

All observations

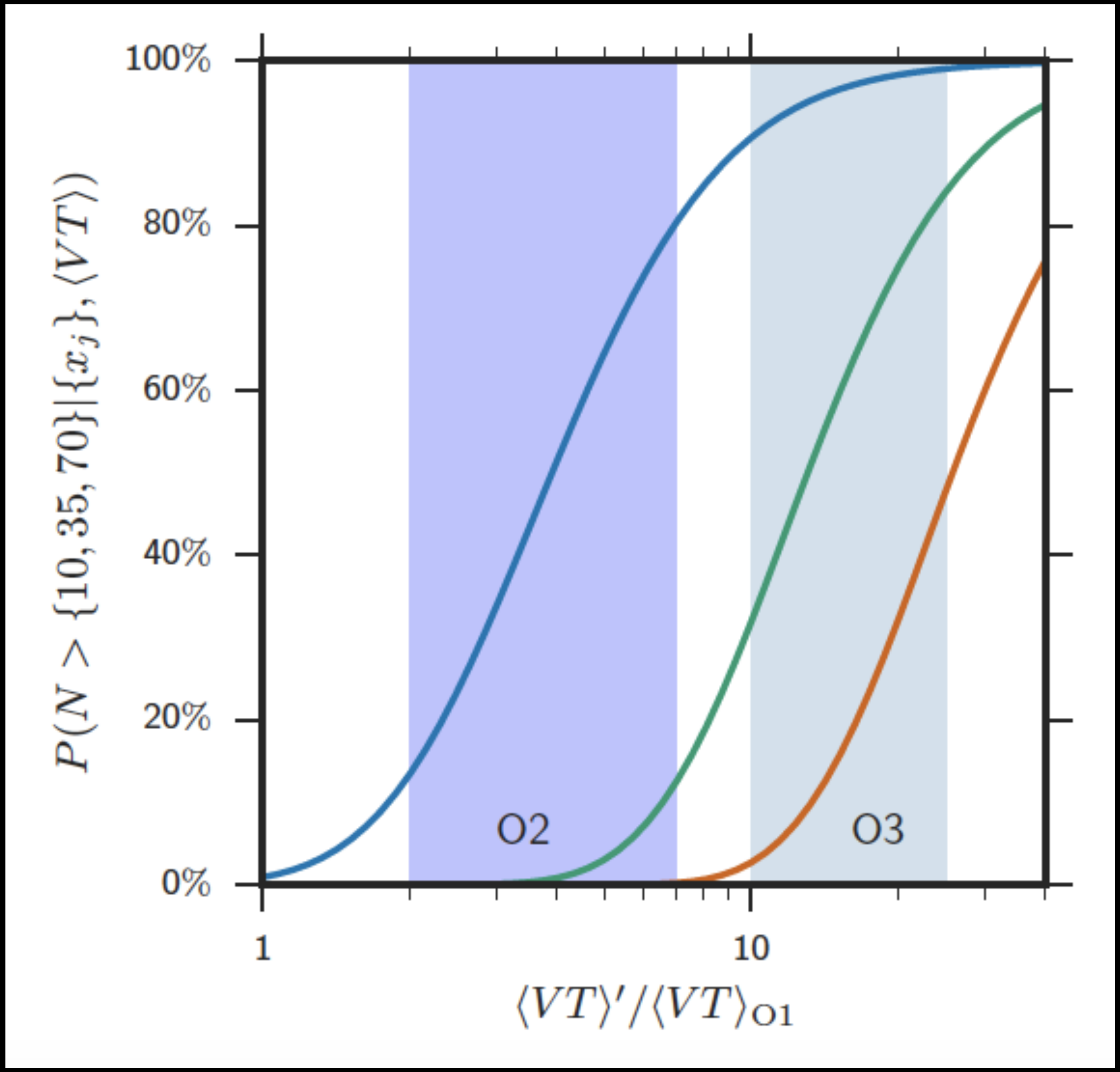


[LVC, arXiv:1606.04856 (2016)]

Black Holes of Known Mass

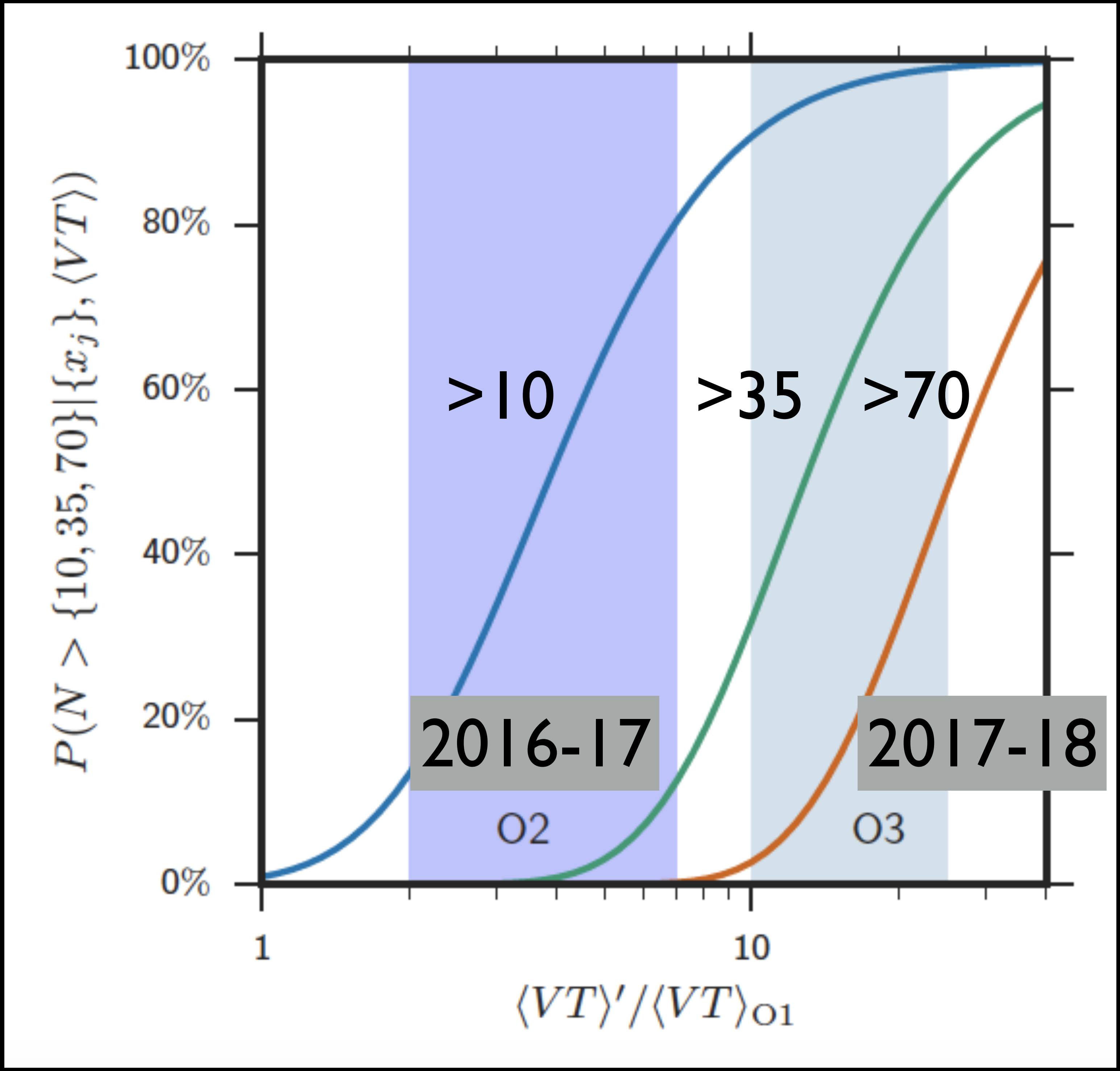


Future observations



[LVC, arXiv:1606.04856 (2016)]

Future observations



[LVC, arXiv:1606.04856 (2016)]

The future

- The field of gravitational-wave astronomy has begun!
- 100s of black hole observations expected in next 5 years

- 2017: Virgo will improve sky sensitivity
- 2020+: Kagra, LIGO India, Einstein Telescope, LIGO Voyager
- 2030+: Space-based detectors