How Might a Black Hole disappear?

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New Frontiers in Black Hole Astrophysics
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Quasi-Normal Modes (QNM)

QNMs.: respond of the black hole to the perturbation

↓

Black hole rings with damped oscillation

↓

QNFs.: carry the characteristic information about the black hole (mass, charge, angular momentum)

↓

Independent of the initial perturbation

Procedure of QNM calculations

- Background metric

{1. Stationary background: Schwarzschild
{2. Time dependent background: Vaidya

perturb it

- Solve the perturbation eq. for appropriate B.C.

$\Psi_{\text{event horizon}} \sim$ pure ingoing wave

$\Psi_{\text{spatial infinity}} \sim$ pure outgoing wave

Procedure of QNM calculations

- Background metric

1. Stationary background: Schwarzschild
2. Time dependent background: Vaidya

perturb it

- Solve the perturbation eq. for appropriate B.C.
- Extract the QNFs from the waveform data

$$\omega = \omega_R + i\omega_I$$

Out-Going Vaidya Metric

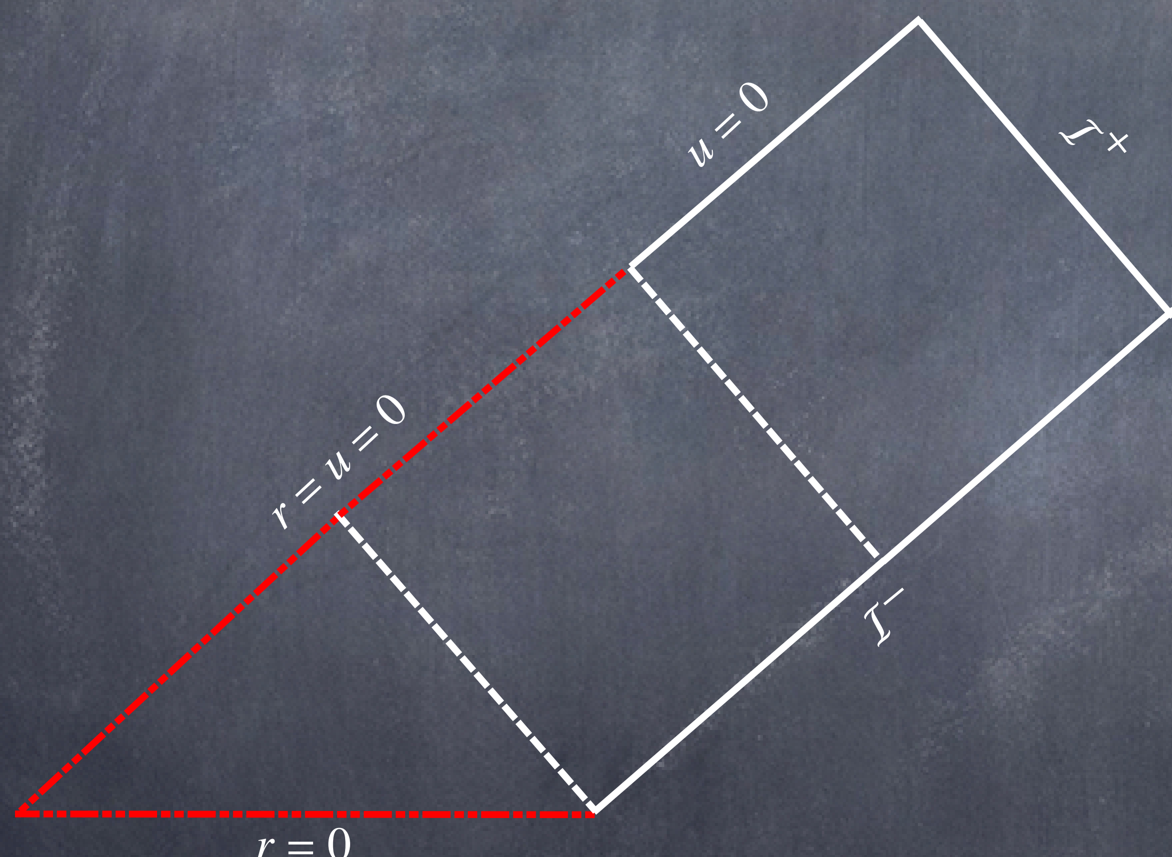
- Out-going Vaidya metric (radiating star & evaporating black hole)

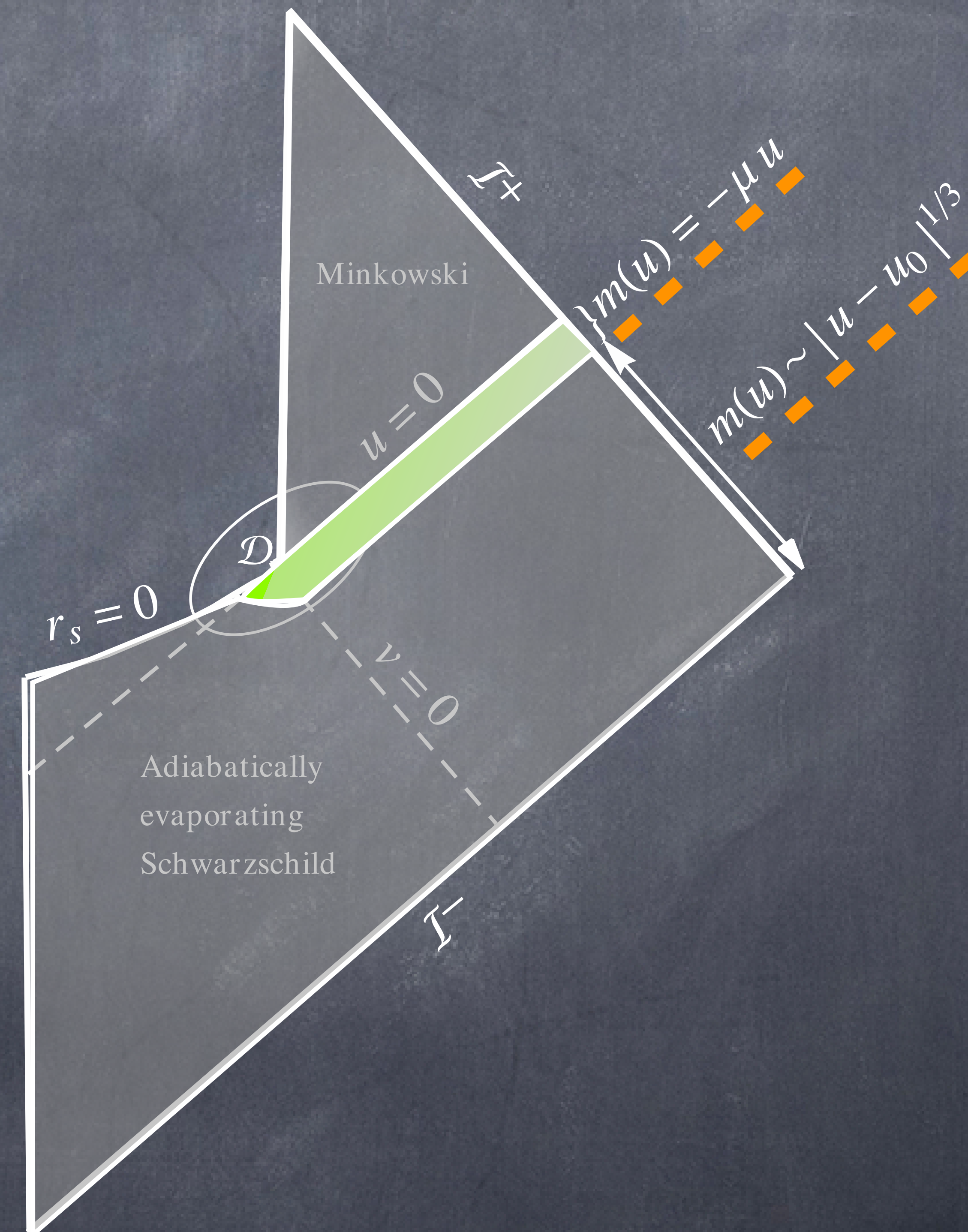
$$ds^2 = -(1 - 2m(u)/r)du^2 - 2dudr + r^2 d\Omega^2$$

- Null coordinates ($u = t-r$ & $v = t+r$)
- Linear mass function $m(u) = -\mu u$
- $\Delta = (1-16\mu)^{1/2}$ $\quad 0 < \mu < 1/16$
- Homothety symmetry

A New Model: Conformal Diagram

$0 < \mu < 1/16$ [1]





Electromagnetic and Scalar Perturbation Equations

$$\frac{\partial^2 \psi(u,v)}{\partial u\,\partial v} + \frac{1+\Delta}{4\Delta\, r(u,v)^4}\left(r(u,v) + \frac{(1-\Delta)}{4}u\right)^{2/(1+\Delta)} \left(\ell(\ell+1)\, r(u,v) - 2\,\sigma\,\mu\, u\right)\psi(u,v) = 0$$

- E.M. perturbation $\sigma = 0$
- Scalar perturbation $\sigma = 1$

- Numerical integration of PDE using double null coordinates method
- Reduction of PDE to ODE ???

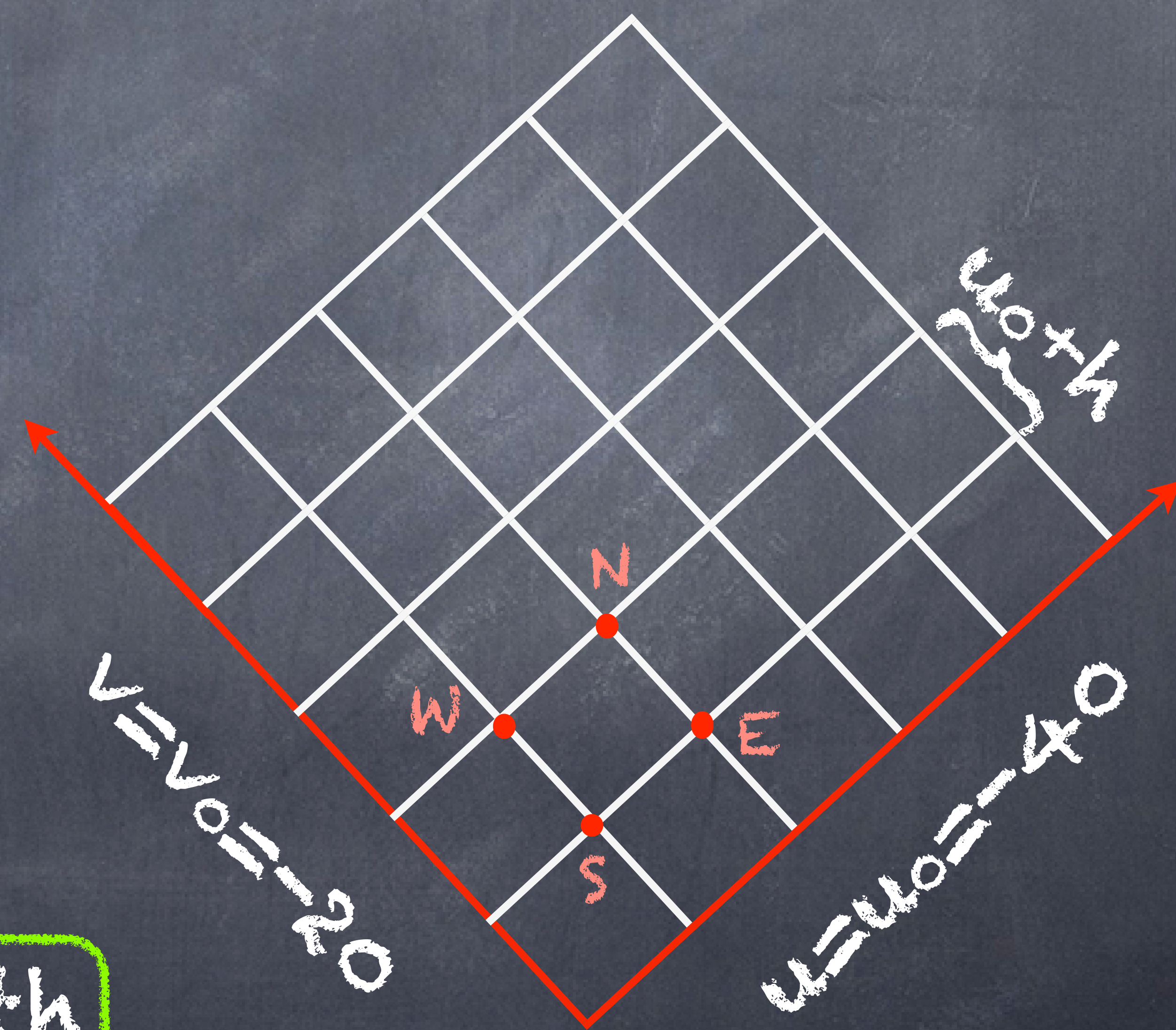
Complicated but possible !!

Double Null Coordinates Integration

Recursion relation [2]:

$$\psi_N = \psi_E + \psi_W - \psi_S - W(u,v)\, h^2/8\, (\psi_E + \psi_W)$$

Potential



Gaussian initial data:

$$\psi(0,v) = \exp[-(v-v_c)^2/w^2]$$

Center

Width

Double Null Coordinates Integration

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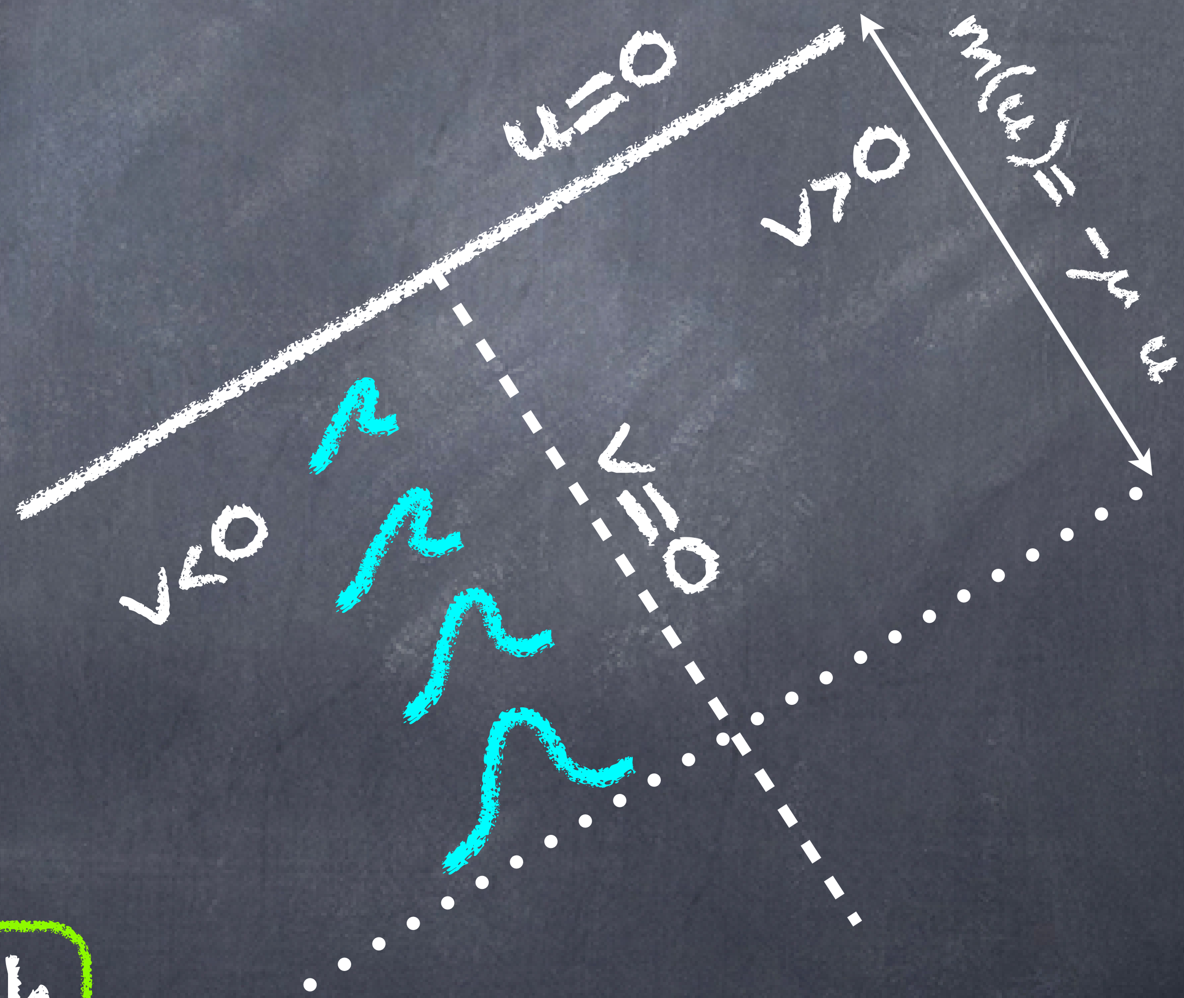
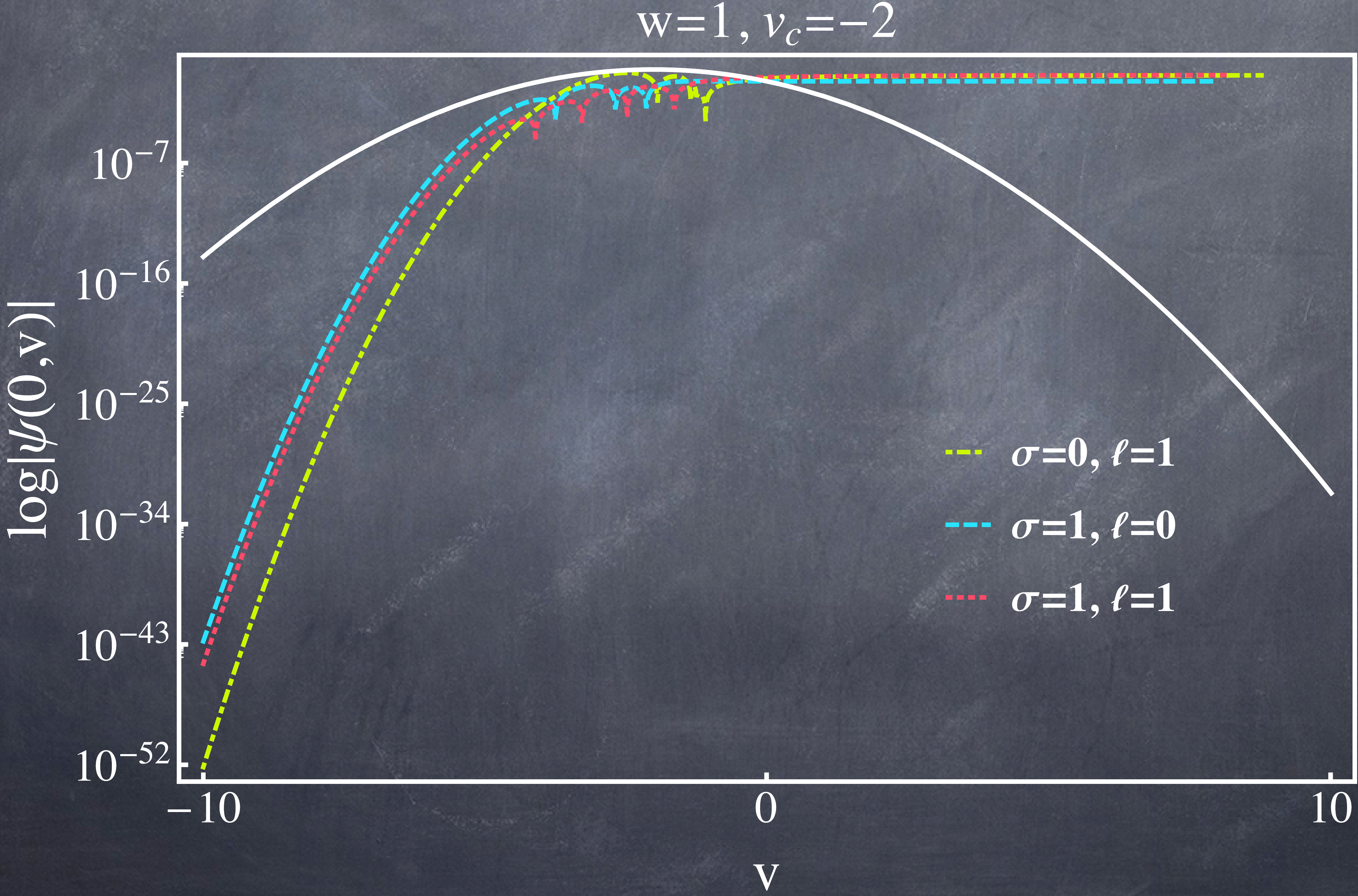
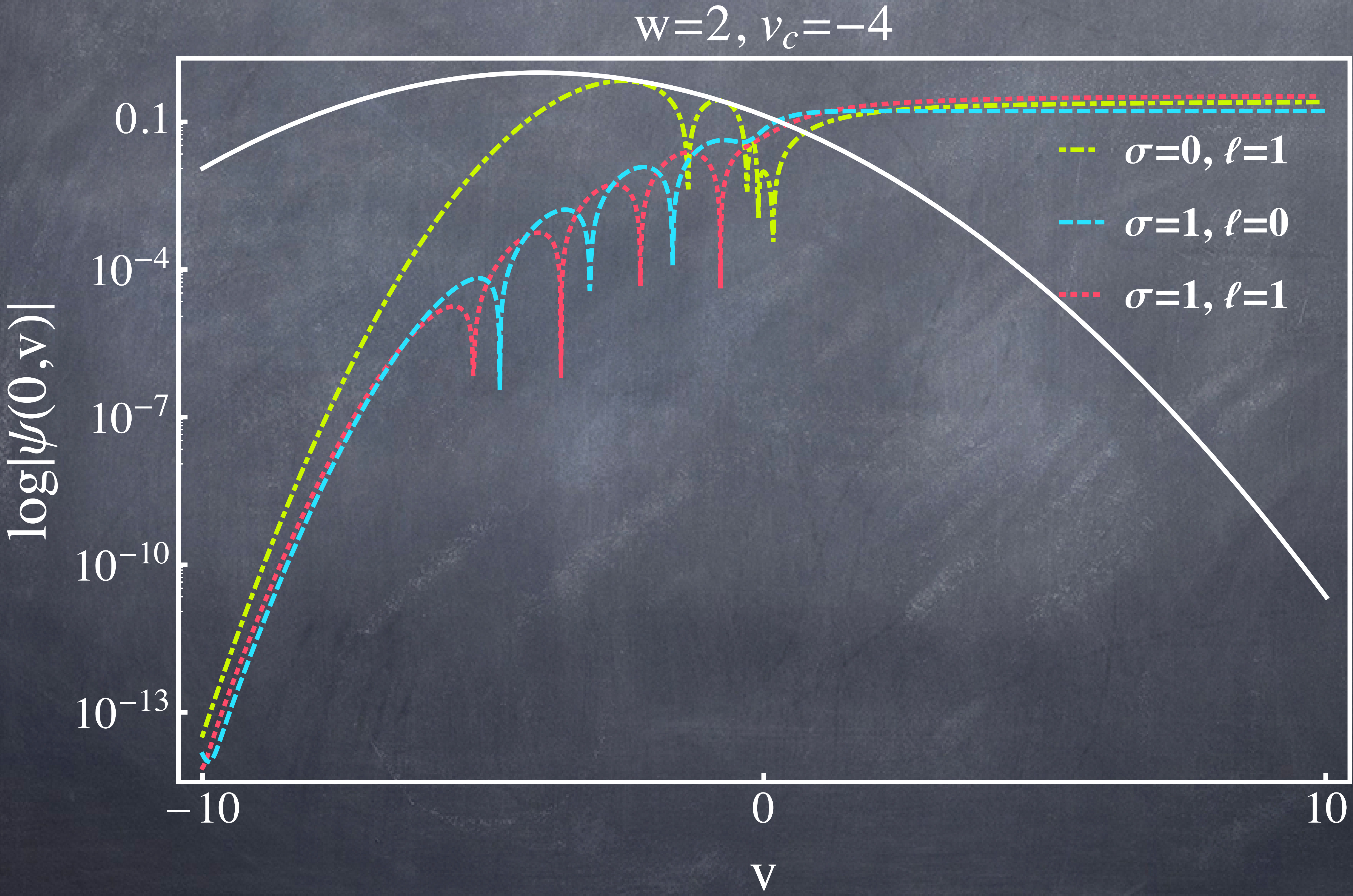


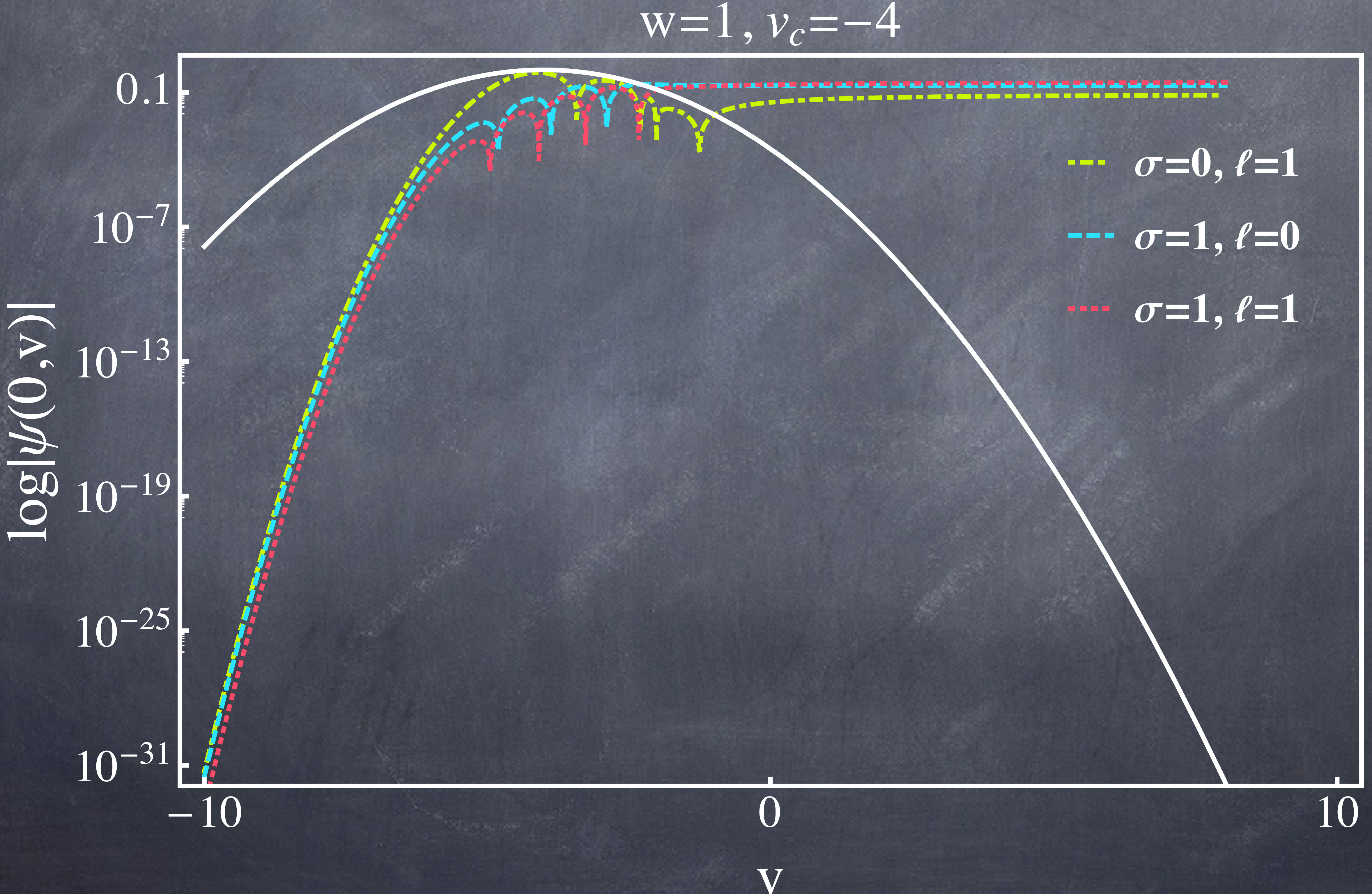
PDE Integration Results for Δ=1/2



PDE Integration Results for $\Delta=1/2$



PDE Integration Results for Δ=1/2



Reduction to an ODE

$$\begin{cases} \bar{u} = (-u) = |u| \\ \bar{v} = v(-u)^{-2\Delta/(1+\Delta)} \end{cases} \longrightarrow \quad r = r(u,v) = |u| g(v/|u|^{2\Delta/(1+\Delta)})$$

Homothetic symmetry $\longrightarrow \quad \psi(\bar{u},\bar{v}) = \bar{u}^{\lambda} V(\bar{v})$

$$\bar{v}\frac{\partial^2 V(\bar{v})}{\partial \bar{v}^2} + (1-\kappa)\frac{\partial V(\bar{v})}{\partial \bar{v}} + F(\bar{v})V(\bar{v}) = 0$$

$$\kappa = \lambda(2\Delta)/(1+\Delta)$$

Analytical Analysis of ODE (Δ=1/2)

Expansion of $F(\bar{v})$ around $\bar{v} \to 0$

$$F(\bar{v}) = \sum_{n=0}^{\infty} b_n \bar{v}^n \longrightarrow \psi_\lambda = \alpha \bar{u}^{2\kappa/3} + \beta v^{\kappa}$$

and around $\bar{v} \to \infty$ and substitution $\bar{v} = e^{\bar{x}}$

$$F(\bar{v}) \approx c\,\ell(\ell+1)/\bar{v}^{5/2} \longrightarrow \psi_\lambda = \gamma \bar{u}^{2\kappa/3} + \delta e^{\kappa x}$$

$\kappa = -i\omega + \varepsilon \qquad \varepsilon \geq 0$

Numerical Solution of ODE

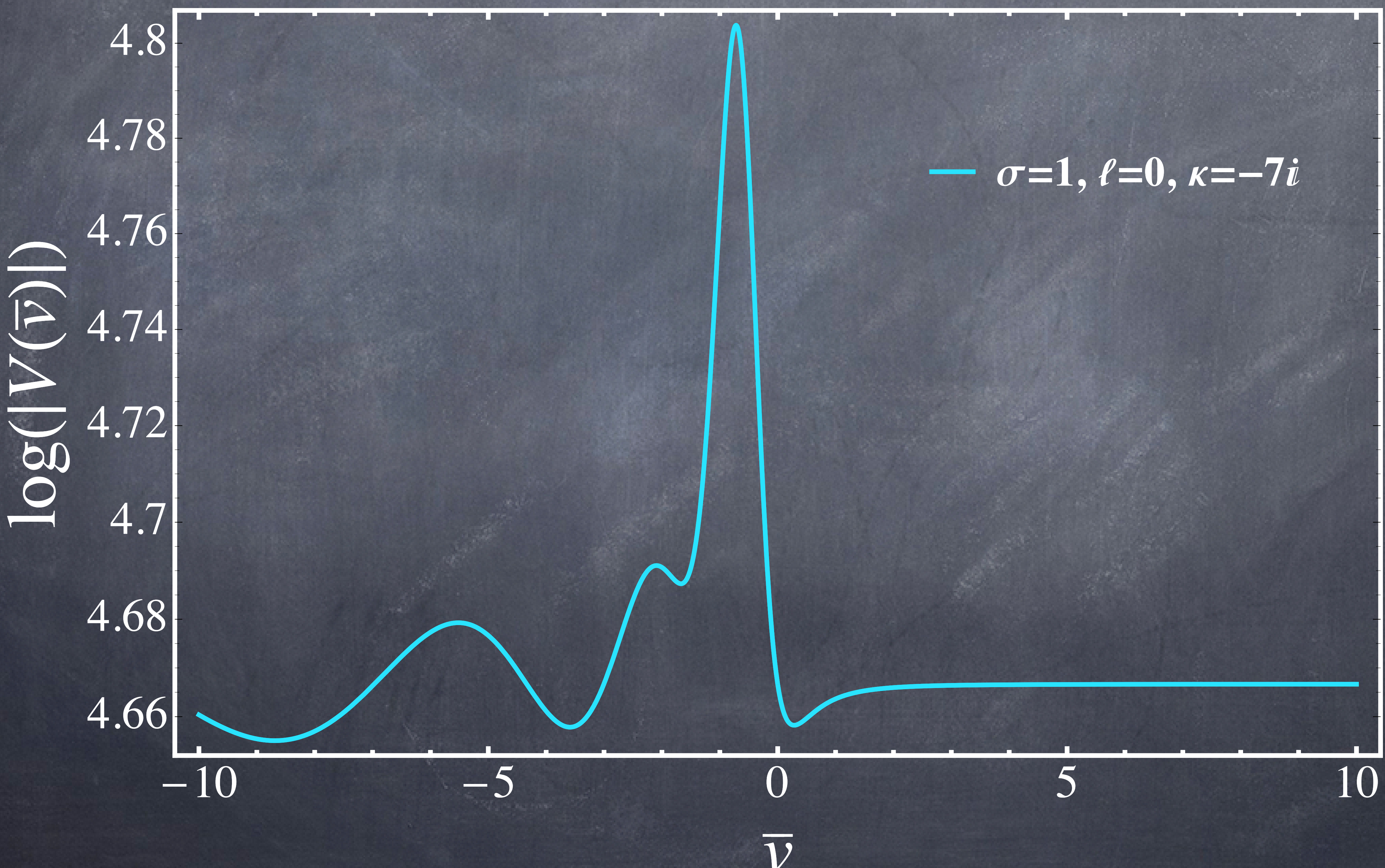
- NDSolve package in Mathematica
- Reasonable initial conditions

→ For $\Delta = 1/2$ solution to p.eq. using LO terms in $F(\bar{v})$

$$V(\bar{v}) \to \bar{v}^{\kappa/2}\left(c_1 J_{-\kappa}\left(8\sqrt[6]{2}\sqrt{\bar{v}}\right) + c_2 J_{\kappa}\left(8\sqrt[6]{2}\sqrt{\bar{v}}\right)\right)$$

- Take the numerical values of the solution for $\bar{v} = 0.000001$ and $\bar{v} = -0.000001$

Results of Numerical Solution of ODE for ε=0



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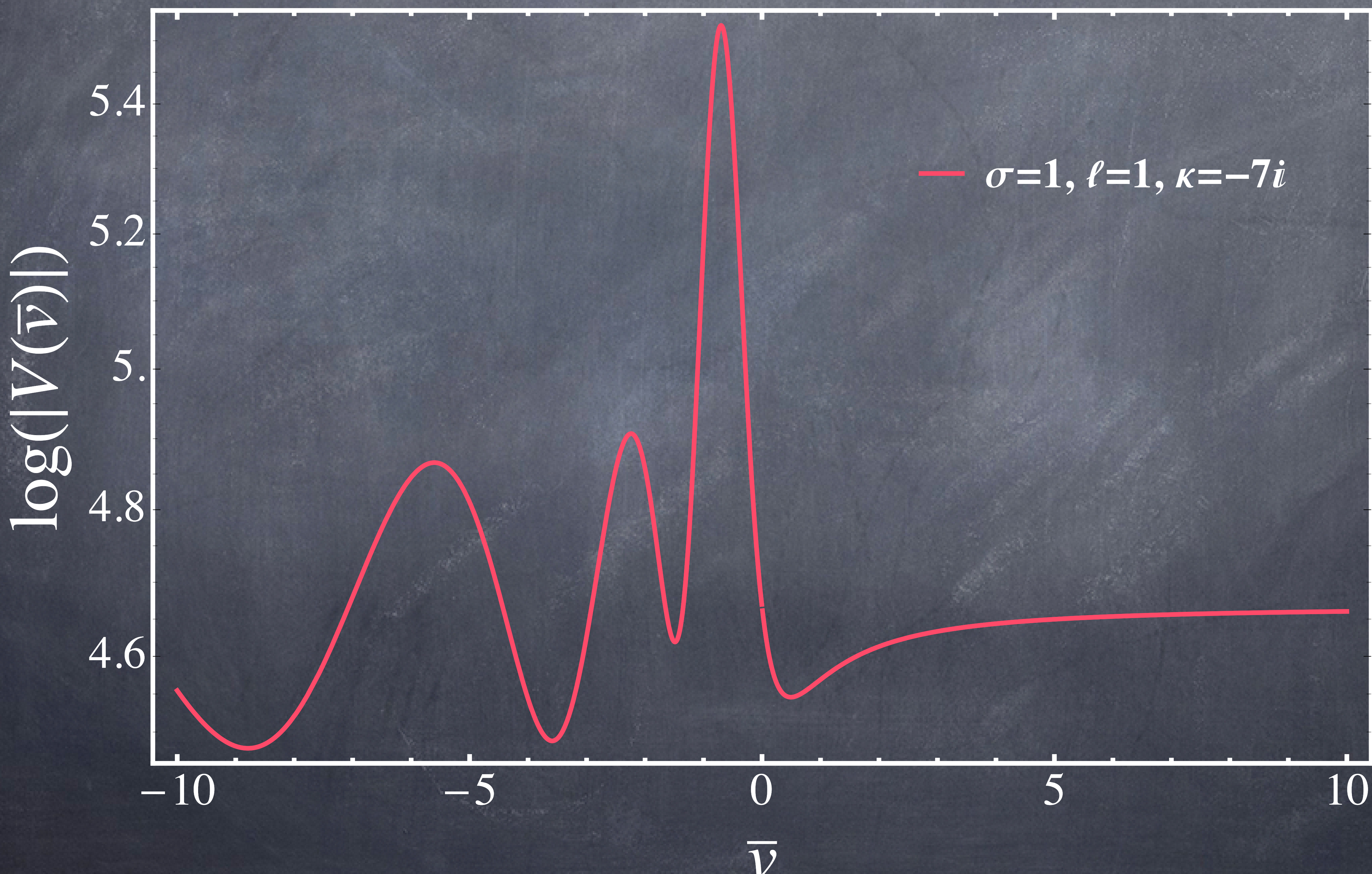

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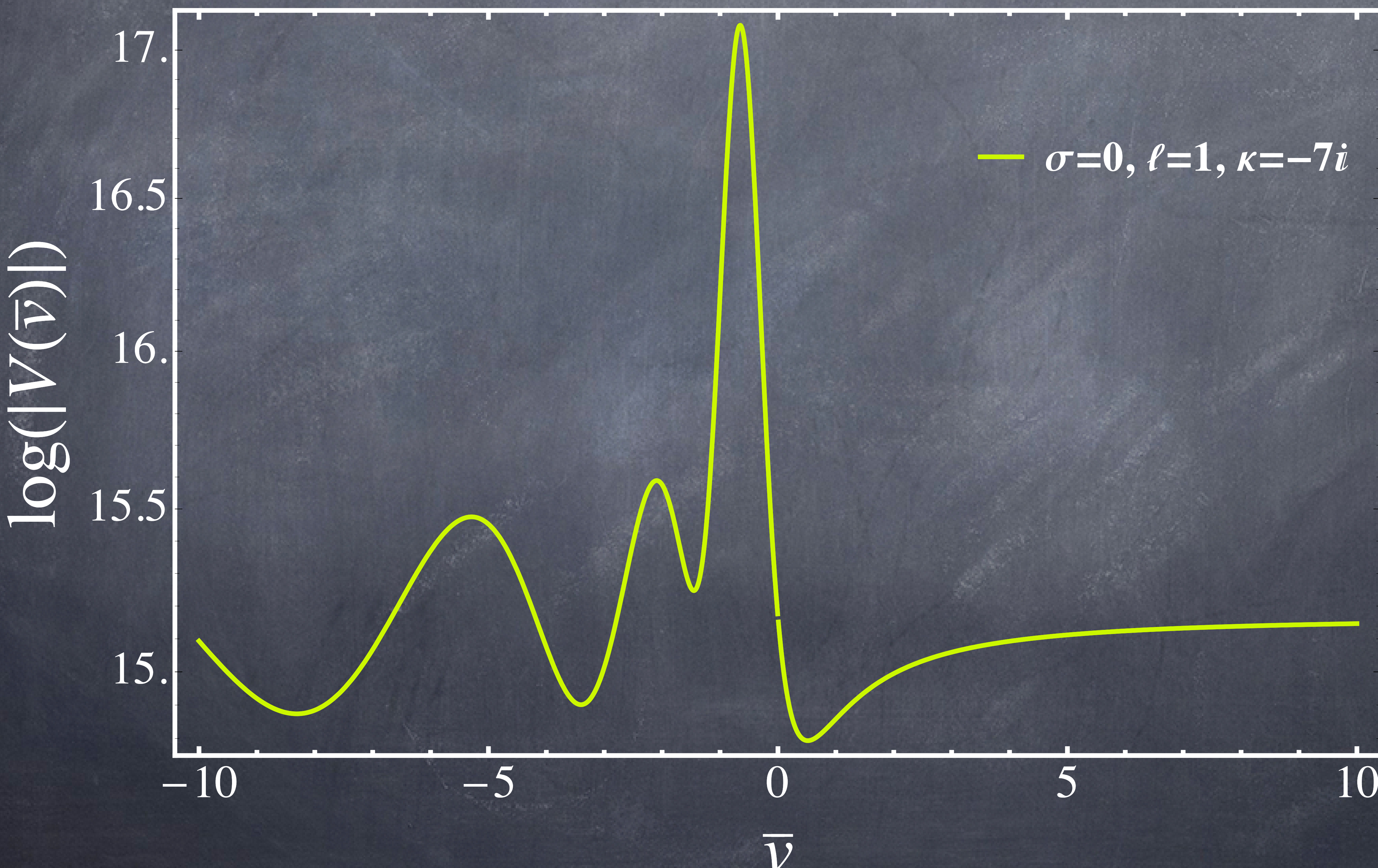

Summary and Conclusions

- Further evidence that outgoing Vaidya space-time may provide a realistic semiclassical model for the end point of black hole evaporation.

- Homothety symmetry → PDE to an ODE → QN-like oscillations around the end point of evaporation.

- The normalizable modes exhibit oscillations as they approach $\bar{u} \to 0$ in both the solutions to the full PDE as well as in the individual modes obtained after separation.

Bibliography

[1] Martin O'Loughlin. A linear mass Vaidya metric at the end of black hole evaporation. Phys. Rev., D91(044020), 2 2015.

[2] Carsten Gundlach, Richard H. Price, and Jorge Pullin. Late time behavior of stellar collapse and explosions: 1. Linearized perturbations. Phys. Rev., D49:883–889, 1994.

[3] Saeede Nafooshe and Martin O'Loughlin. Wave equations on the linear mass Vaidya metric. Phys. Rev., D94(044035), 2016.