Post-Newtonian-accurate regularized SMBH dynamics in galaxy simulations

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From galaxy mergers to GW coalescence

Evolutionary phase	Distance scale (approximate)
Galaxies in group/cluster environment	1 kpc – 1 Mpc
Galaxy mergers	0.1 kpc – 100 kpc
Dynamical friction	10 pc – 1 kpc
Binary hardening by three-body scatterings	0.01 pc – 10 pc
GW emission, SMBH merger	AU scale – 0.01 pc

Tree-gravity/hydro codes

Direct summation codes

Few-body PN codes

Numerical relativity

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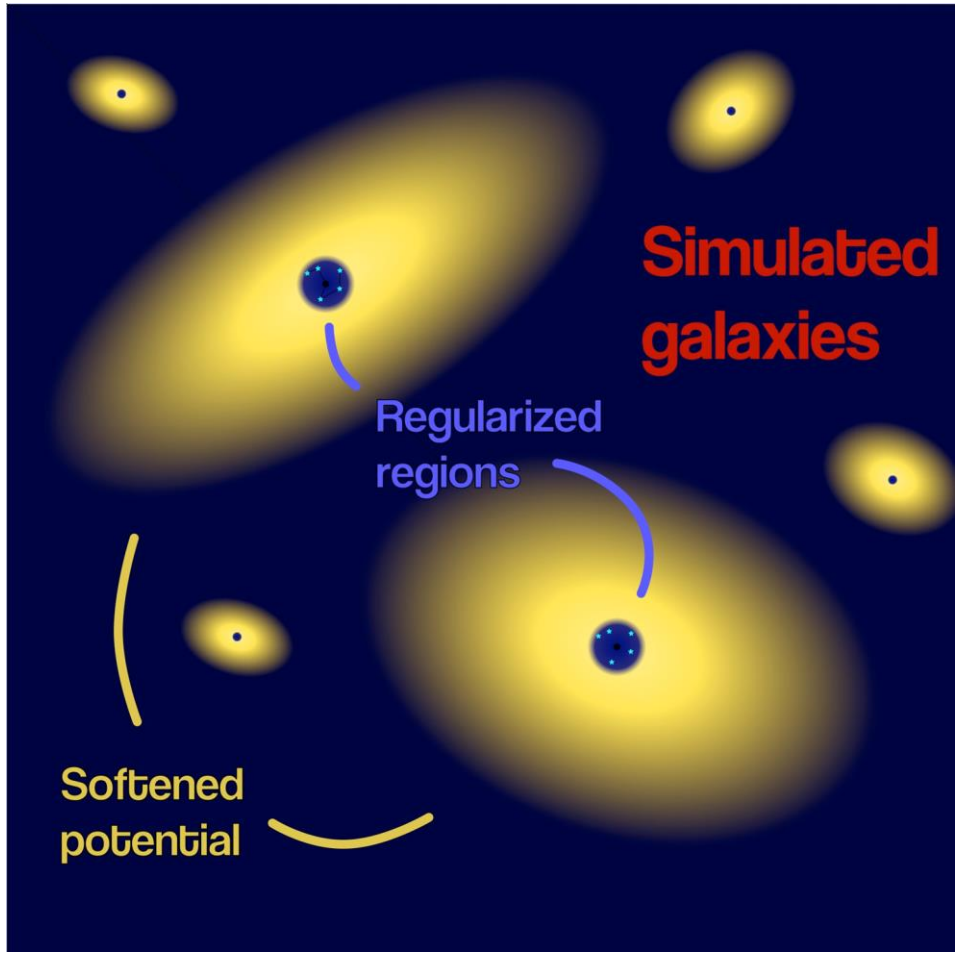
Our focus

KETJU: regularized SMBH dynamics in Gadget-3



Gadget-3:

- Softened Newtonian gravity with TreePM algorithm
- Gas dynamics using a modern Smoothed Particle Hydrodynamics
- Sub-resolution star-formation, stellar feedback, SMBH accretion+feedback, metals, metal-dependent cooling...

KETJU:

- A regularized volume around the SMBHs
- Accurate, non-softened dynamics
- Post-Newtonian corrections up to PN3.5, optional spin- dependent terms and their cross terms
- PN approximation accurate down to approximatively 10 Schwarszchild radii of the SMBHs

KETJU (Finnish):
A chain

Algorithmic Chain Regularization (ARCHAIN)

- The equations of motion are time-transformed. Together with a leapfrop integrator, this regularizes the system against Newtonian force divergences.
- Chain: the usage of chained inter-particle vectors significantly reduces the round-off error.
- Bulirsch-Stoer extrapolation method to formally extrapolate dt → 0. This corresponds to taking a large number of substeps during one Gadget-3 timestep.
- Error in dynamical variables of the chain particles can be pushed down to machine precision.

Define $t \mapsto s$ by

$$\begin{aligned} \mathrm{d}s &= [\alpha(T+B) + \beta\omega + \gamma]\,\mathrm{d}t \\ &= (\alpha U + \beta\Omega + \gamma)\,\mathrm{d}t, \end{aligned}$$

where $\alpha, \beta, \gamma \in \mathbb{R}$, and

$$T = \sum_i \frac{1}{2} m_i \|\vec{v}_i\|^2 \quad \text{kinetic energy},$$

$$U = \sum_i \sum_{j>i} \frac{G m_i m_j}{\|\vec{r}_{ij}\|} \quad \text{force function},$$

$$B = -T + U \quad \text{binding energy},$$

$$\Omega = \text{arbitrary function of } \vec{r}_i,$$

$$\dot{\omega} = \sum_i \nabla_{\vec{r}_i} \Omega \cdot \vec{v}_i.$$

Chain subsystems in Gadget-3

- **Chain particles**

  SMBHs and stars inside the influence radius.

- **Tree particles**

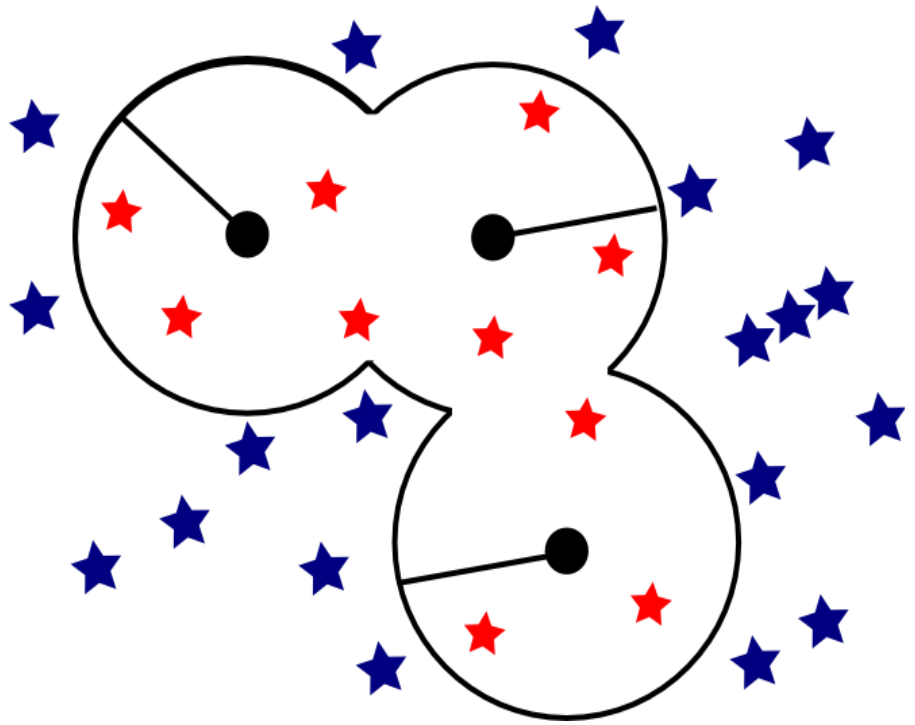
  Ordinary Gadget-3 particles.

- **Perturber particles**

  Tree particles strongly perturbing a chain subsystem.

  User-defined parameter lambda and gamma set the amount of chain and perturber particles.

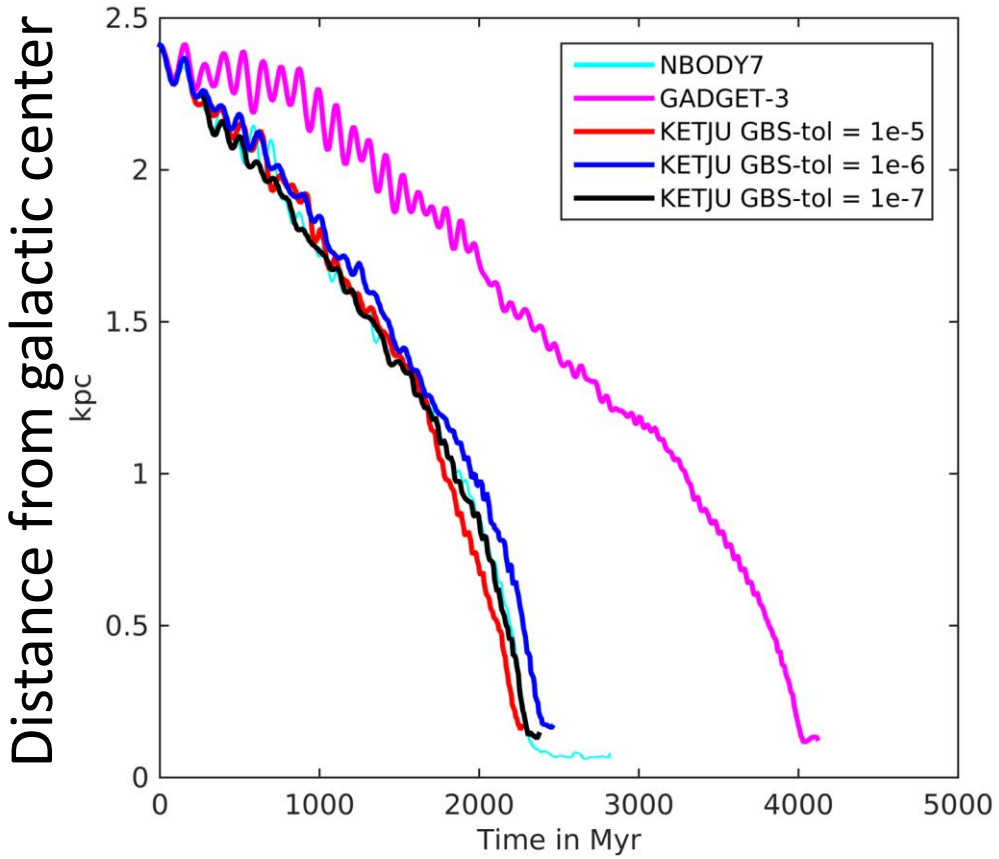
$$r_{\mathrm{infl}} = \lambda \times \frac{M_{\mathrm{BH}}}{10^{10} M_{\odot}} \mathrm{kpc}$$



Chain & Tree memberships updated every timestep

$$r < r_{\mathrm{pert}} = \gamma \times r_{\mathrm{infl}} \left(\frac{m}{M_{\mathrm{BH}}} \right)^{1/3}$$

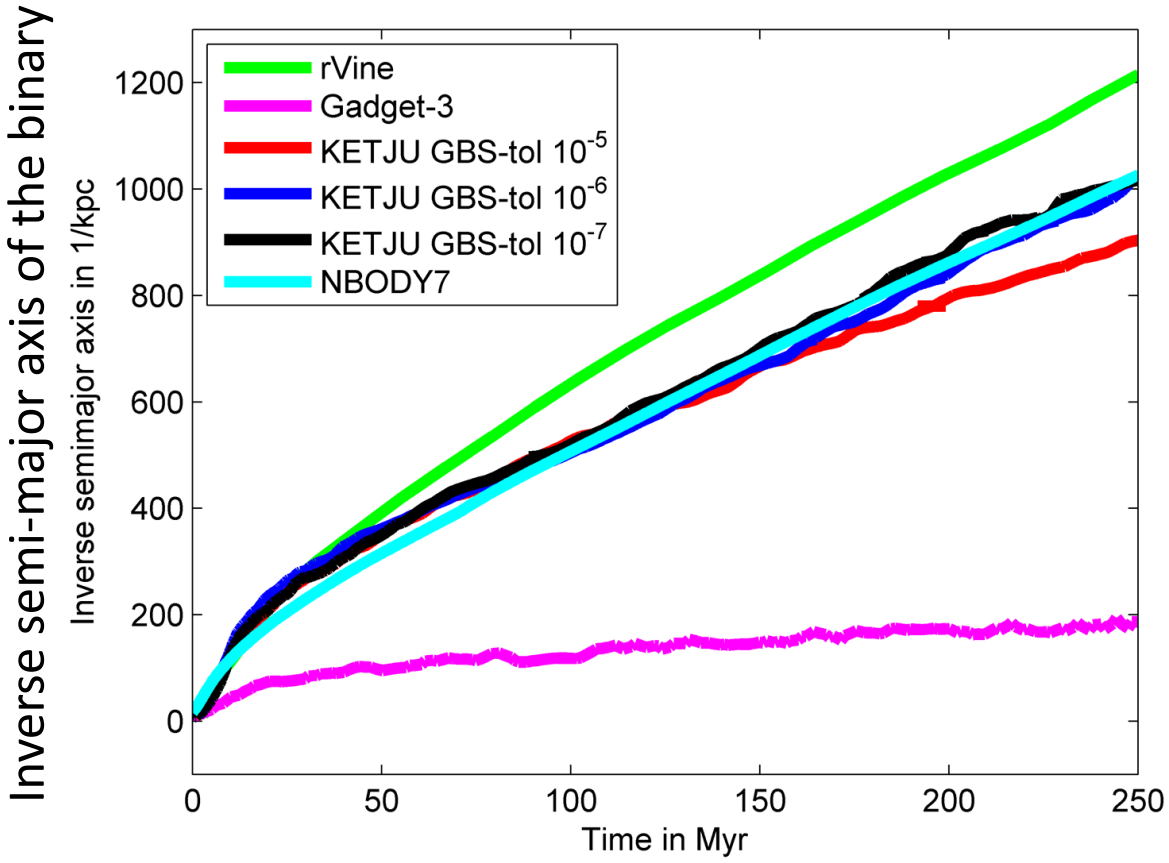
Comparing KETJU to ordinary Gadget-3 and NBODY7



Time

A single SMBH sinking due to dynamical friction in a Hernquist sphere

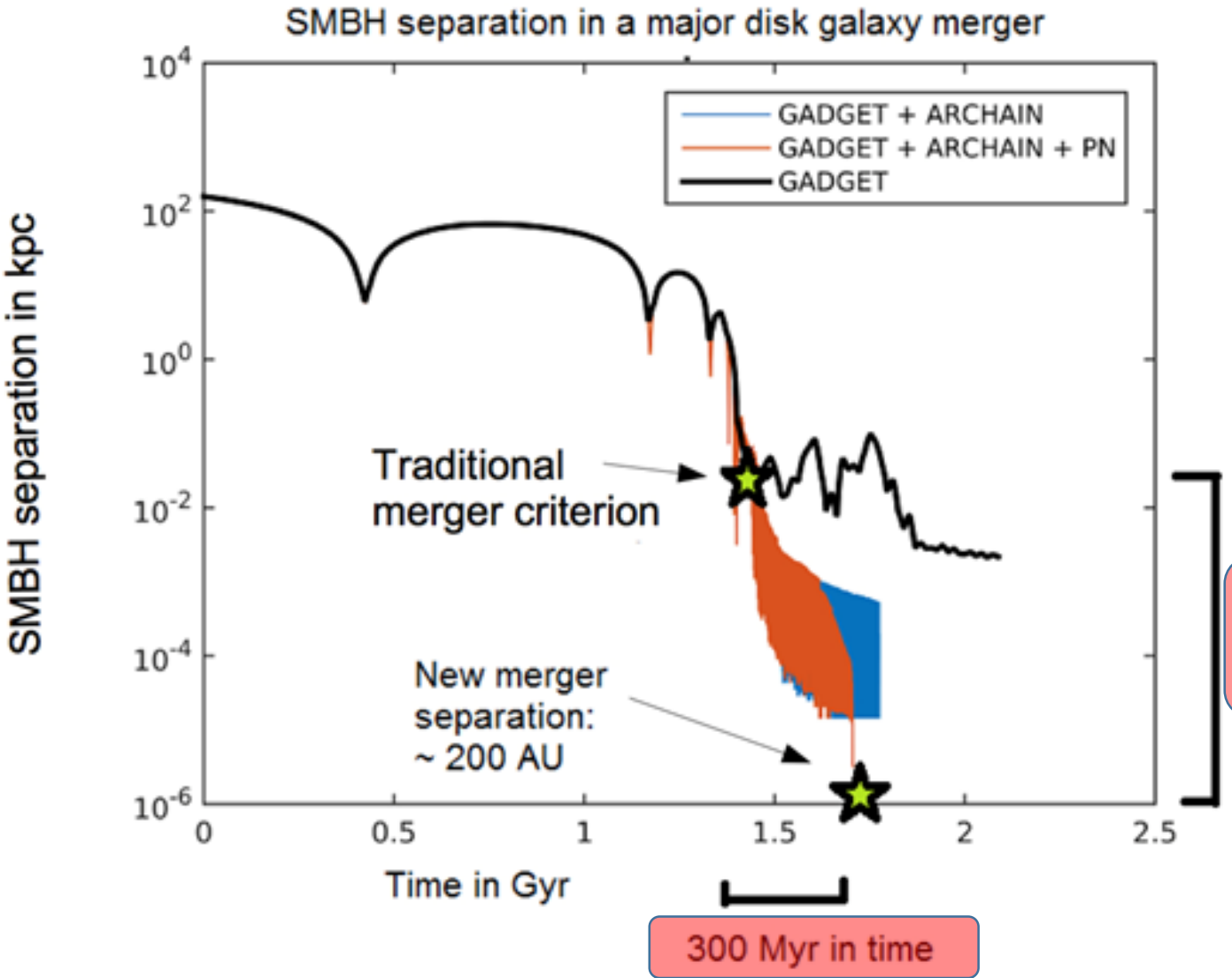
GBS-tol = Bulirsch-Stoer integrator accuracy



Time

A SMBH binary hardening via 3-body interactions of stars

Realistic SMBH merger timescales Gadget-3 – like codes



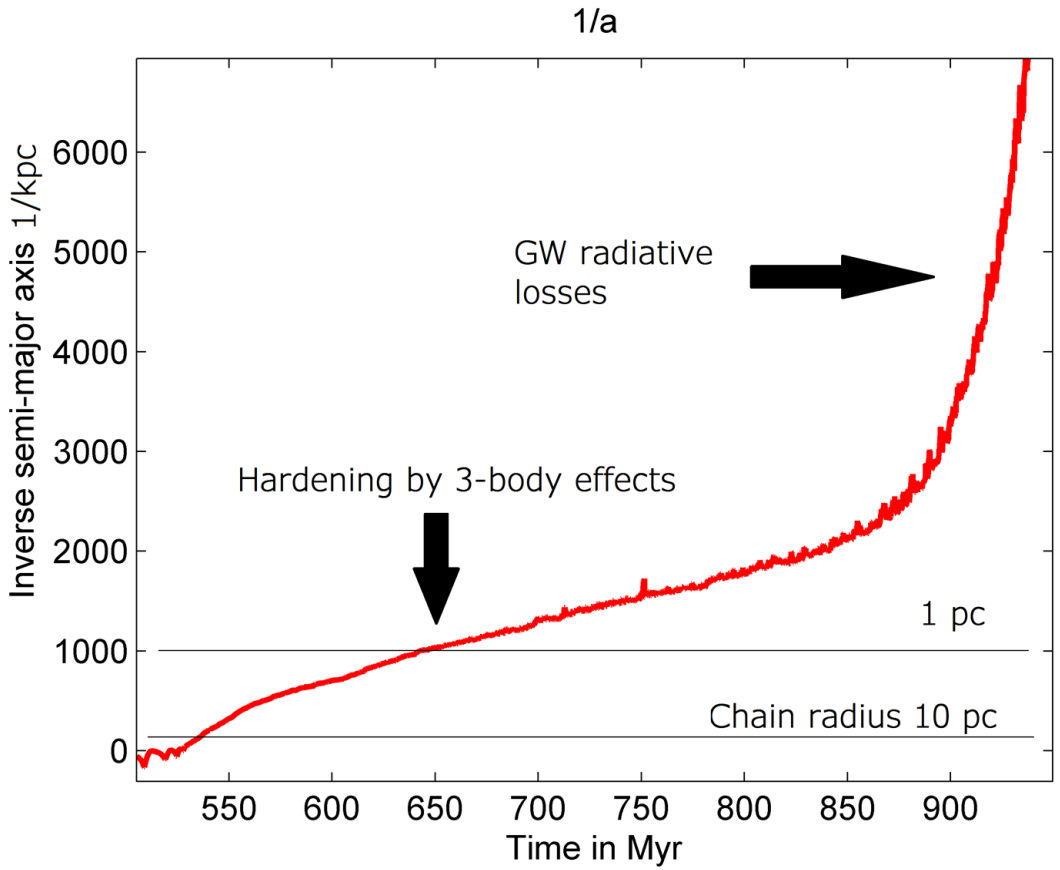
The original Gadget-3 merger criterion: merge SMBHs instantly when their softening lengths overlap and the relative velocity is small enough

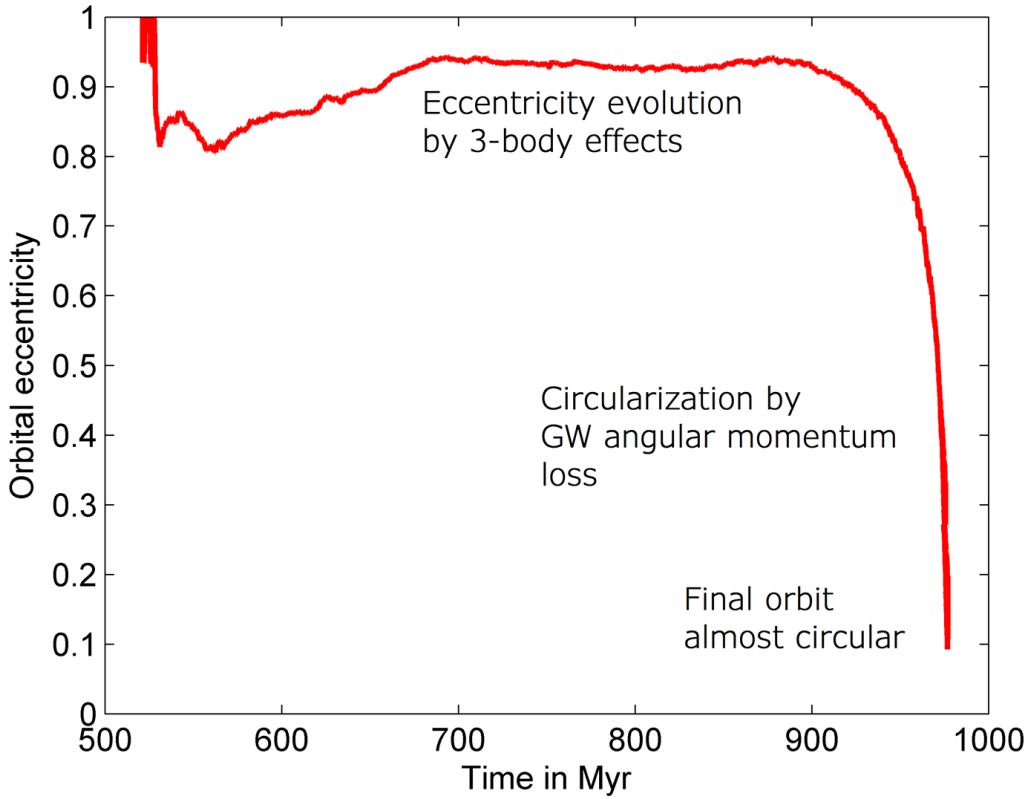
New KETJU criterion is based on the GW coalescence timescale of the binary obtained from Peter's formula (1964)

$$t_c \sim \frac{a}{4\dot{a}}$$ < Gadget timestep

$$\left|\frac{da}{dt}\right| = \frac{64}{5}\frac{G^3 M_1 M_2 (M_1+M_2)}{c^5 a^3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}$$

High-resolution dry mergers of elliptical galaxies with SMBHs using KETJU (Rantala et al. to be submitted)





High numerical resolution extremely important!

Summary

- We have developed KETJU, a regularized dynamics module for Gadget-3.

- More accurate SMBH merger timescale estimates.

- Next step: KETJU + hydrodynamics + subresolution feedback

  → PN3.5-accurate SMBH dynamics

  + galactic-scale hydrodynamics *using a single simulation code.*