Ray-tracing and polarized radiation transfer in General Relativity – Introducing the ARCMANCER code

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Section 1

Motivation – why another ray-tracing code?

Why develop ARCMANCER?

Good qualities for any scientific code

- Open source: freely downloadable in source code format.
- Clear and well written code base, conforming to existing coding standards.
- Well documented and easy to use (e.g. a Python interface (API) would be *really* useful).

Good qualities for a GR ray-tracing code

- Support for arbitrary metrics and coordinate charts (e.g. not just Kerr metric).
- Support for polarized radiation transfer (PRT) computations.

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Why develop ARCMANCER?

Code	Open source	PRT	Arb. metrics	Python API
GYOTO	Yes	No	Yes	No
GEOKERR	Yes	No	No	No
GRTRANS	Yes	Yes	No	Yes
GRAY	Yes	No	No	No
KERTAP	Yes	No	No	Yes
BB2004[1]	No	Yes	No	?
ASTRORAY	Yes	Yes	No	No
ARCMANCER	Yes	Yes	Yes	Yes

[1] Broderick & Blandford (2004)

Why develop ARCMANCER?

Polarized X-ray emission from compact objects

- Black hole accretion disks, jets (Kerr metric ok)
- Accreting millisecond pulsars (Kerr metric *not* ok)
- Multiple black hole systems (numerical/approximate metrics)

Recent spaceborne X-ray polarimeter proposals

Mission	Organization	Status
PRAXyS	NASA	SMEX[a] Phase A assessment
IXPE	NASA	SMEX Phase A assessment
XIPE	ESA	M4 Phase A assessment
XTP	CNSA[b]	Expected launch 2020–2025
eXTP	CNSA + LOFT (ESA)	Aiming for pre-2025 launch

[a] Small Explorers
[b] Chinese National Space Administration

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Section 2

Polarized radiation transfer with ray-tracing

Why ray-tracing and what is it good for?

Why ray-tracing?

- When spacetime is curved, photons propagate on curved paths.
- No analytic solutions when spacetime lacks symmetry.

What is radiation transfer via ray-tracing?

- Numerically propagate photon paths.
- Then do radiation transfer along those paths.
- Get relative Doppler shifts, gravitational redshift & Faraday rotation etc. for free.

Polarization and GR

Polarization

- Polarization: components of the $\boldsymbol{E}$-field in a plane orthogonal to photon propagation direction $\boldsymbol{k}$ (*polarization plane*).
- In GR, polarization plane orthogonal to k^a *and* observer four-velocity $u^a \Rightarrow$ observer dependent polarization

Alternative approaches (Gammie & Leung 2012)

- Define polarization frame in plasma rest frame at each point.
- Directly propagate polarization tensor.
- Define polarization frame at source, parallel transport to observer.
- Define polarization frame at observer, parallel transport to source.

Radiation transfer

In flat space

- $\dfrac{\mathrm{d}\boldsymbol{I}_\nu}{\mathrm{d}s} = \boldsymbol{J}_\nu - \boldsymbol{M}\boldsymbol{I}_\nu$
- $\boldsymbol{I} = (I, Q, U, V)$ is the Stokes vector
- $\boldsymbol{J} = (j_I, j_Q, j_U, j_V)$ is the emissivity vector
- absorption and Faraday rotation/conversion given via the response (Müller) matrix

$$\boldsymbol{M} = \begin{pmatrix} \alpha_I & \alpha_Q & \alpha_U & \alpha_V \\ \alpha_Q & \alpha_I & r_V & -r_U \\ \alpha_U & -r_V & \alpha_I & r_Q \\ \alpha_V & r_U & -r_Q & \alpha_I \end{pmatrix}$$

Radiation transfer

In curved space

Project polarization frame to local medium rest frame and solve

$$\frac{\mathrm{d}\boldsymbol{\mathcal{I}}_{\nu_0}}{\mathrm{d}\lambda} = \boldsymbol{\mathcal{J}}_{\nu_0} - \boldsymbol{\mathcal{M}}_{\nu_0}\boldsymbol{\mathcal{I}}_{\nu_0},$$ the relativistic PRT equation,

where ν_0 is the *observed* frequency and we have the relativistic

$$\boldsymbol{\mathcal{I}}_{\nu_0} = (\mathcal{G}/\nu_0)^3 \boldsymbol{I}_{\mathcal{G}^{-1}\nu_0} \in \mathbb{R}^4$$ Stokes intensities

$$\boldsymbol{\mathcal{J}}_{\nu_0} = (\mathcal{G}/\nu_0)^2 \boldsymbol{J}_{\mathcal{G}^{-1}\nu_0} \in \mathbb{R}^4$$ Stokes emissivities

$$\boldsymbol{\mathcal{M}}_{\nu_0} = (\mathcal{G}/\nu_0)^{-1} \boldsymbol{M}_{\mathcal{G}^{-1}\nu_0} \in \mathbb{R}^{4\times 4}$$ response matrix,

where the redshift factor (gravitational, cosmological & Doppler) is

$$\mathcal{G} = \frac{\nu_0}{\nu} = \frac{{u_0}^a p_{0,a}}{u^a p_a} = \frac{\text{observer 4-velocity} \cdot \text{photon 4-momentum @ obs.}}{\text{source 4-velocity} \cdot \text{photon 4-momentum @ source}}.$$

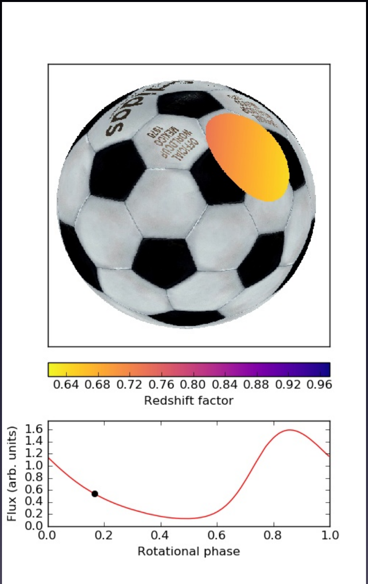
Section 3

First applications

Subsection 1

Rotating neutron stars

Imaging & animations



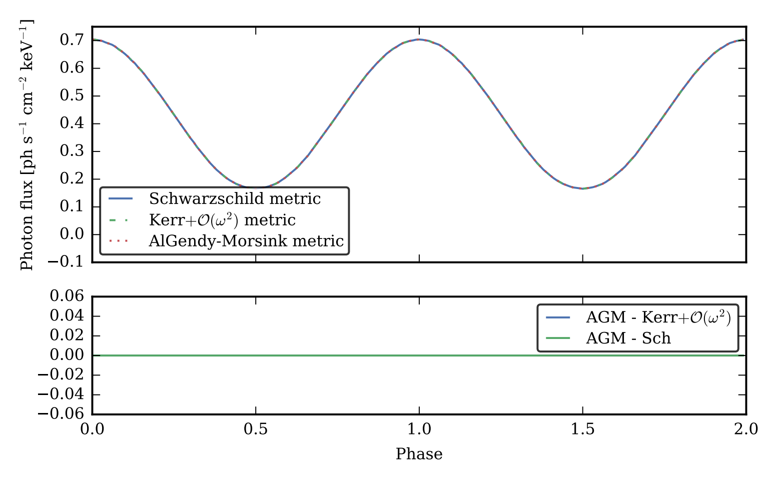
Ray-traced image of a rotating neutron star (NS) with an accretion hotspot, rotating at 700 Hz.

Interesting features

- Both poles of the star are visible
- Surface distorted by time-delay effects
- Varying redshift factor as the spot rotates
- Strongly non-sinusoidal flux curve

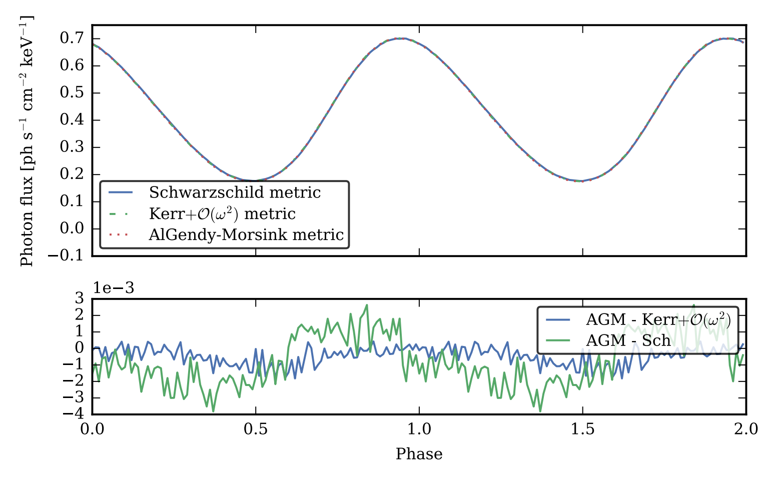
Accreting NS at $\sim$0 Hz

Figure : NS with $M = 1.4 M_\odot$, $R = 15$ km. Observer inclination 45°. Hotspot $T = 2$ keV, colatitude 45°, angular diameter 30°. Distance 10 kpc.



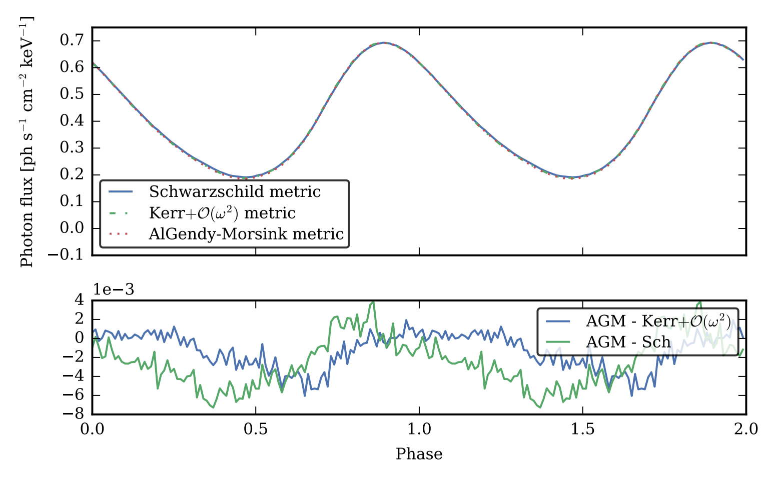
Accreting NS at 250 Hz

Figure : NS with $M = 1.4 M_\odot$, $R = 15\,$km. Observer inclination $45°$. Hotspot $T = 2\,$keV, colatitude $45°$, angular diameter $30°$. Distance 10 kpc.



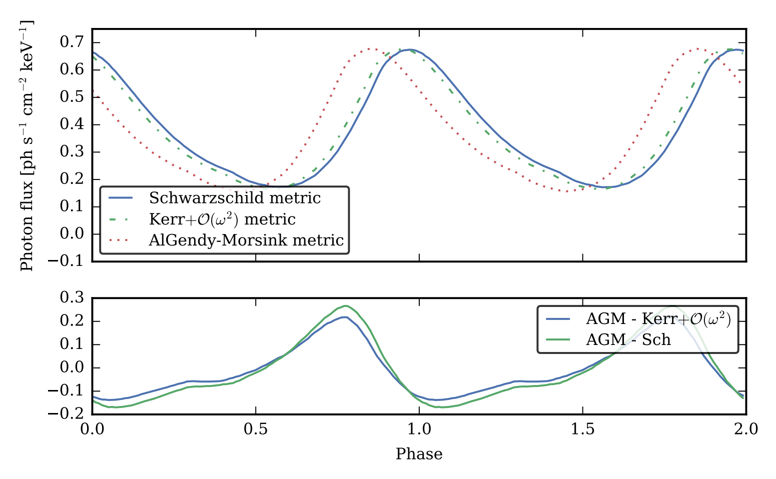
Accreting NS at 500 Hz

Figure : NS with $M = 1.4 M_{\odot}$, $R = 15$ km. Observer inclination $45°$. Hotspot $T = 2$ keV, colatitude $45°$, angular diameter $30°$. Distance 10 kpc.



Accreting NS at 750 Hz

Figure : NS with $M = 1.4 M_\odot$, $R = 15\,$km. Observer inclination 45°. Hotspot $T = 2\,$keV, colatitude 45°, angular diameter 30°. Distance 10 kpc.

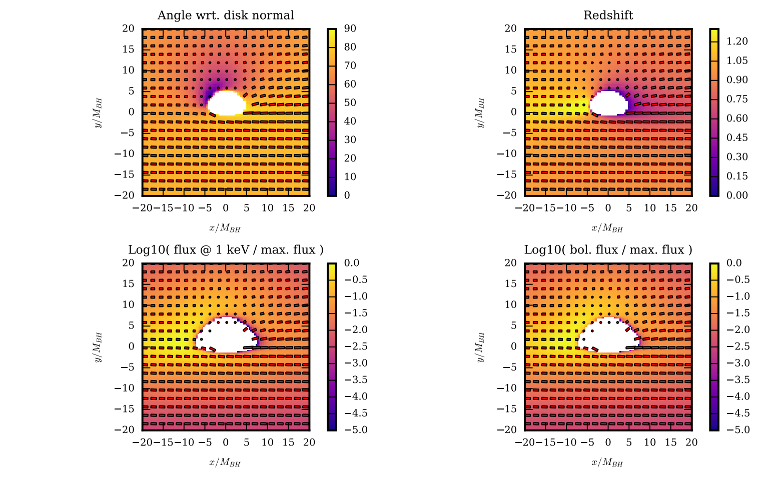


Subsection 2

Black holes and accretion disks

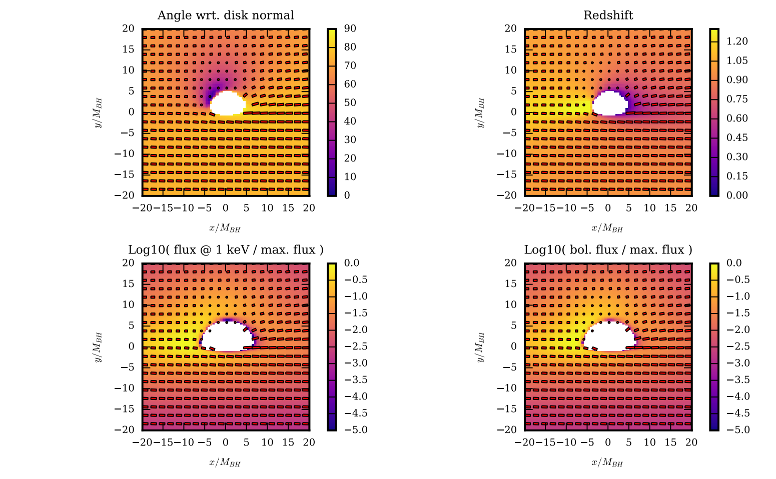
BH non-dimensional spin $\chi = 0$

Figure : Black hole with $M = 10M_\odot$, $\dot{M} = 0.1M_{\text{Edd}}$. Novikov–Thorne disk with $\alpha = 0.1$. Observer inclination $75°$.



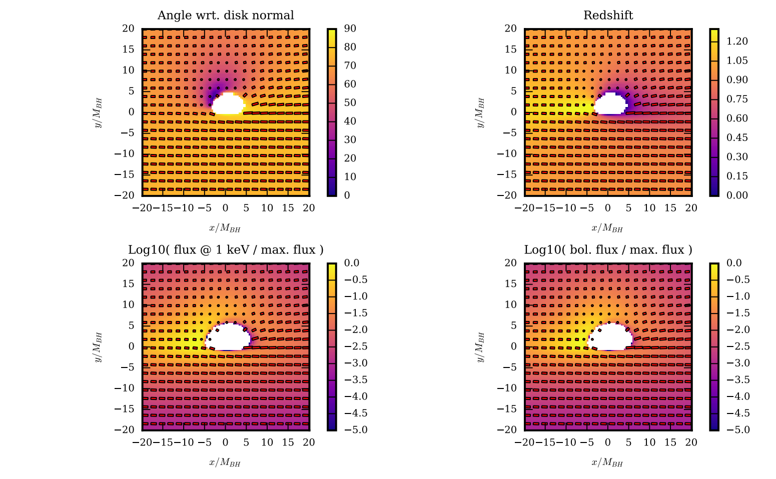
BH non-dimensional spin $\chi = 0.3$

Figure : Black hole with $M = 10M_\odot$, $\dot{M} = 0.1M_{\text{Edd}}$. Novikov–Thorne disk with $\alpha = 0.1$. Observer inclination $75°$.



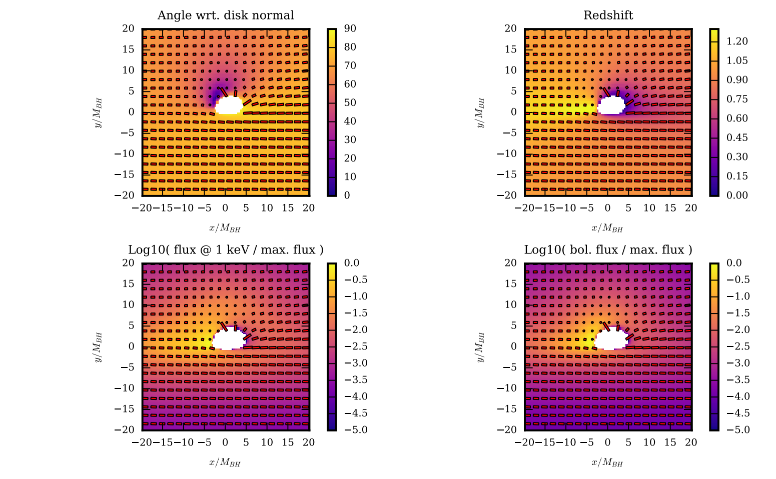
BH non-dimensional spin $\chi = 0.6$

Figure : Black hole with $M = 10M_{\odot}$, $\dot{M} = 0.1M_{\text{Edd}}$. Novikov–Thorne disk with $\alpha = 0.1$. Observer inclination $75°$.



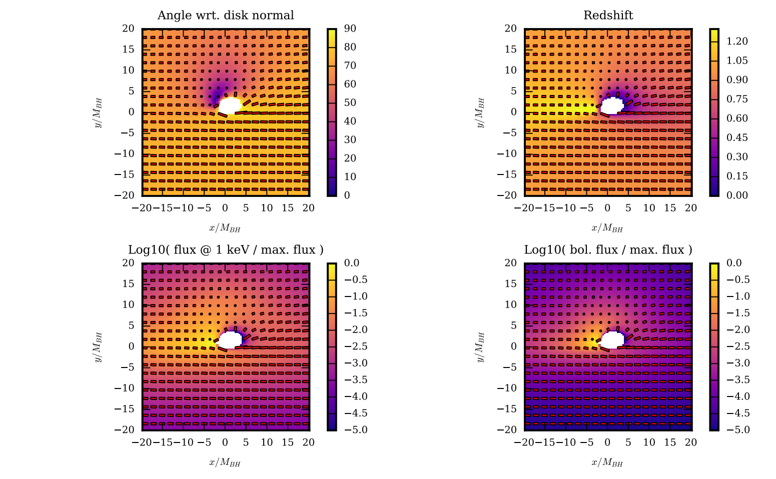
BH non-dimensional spin $\chi = 0.9$

Figure : Black hole with $M = 10M_\odot$, $\dot{M} = 0.1M_{\text{Edd}}$. Novikov–Thorne disk with $\alpha = 0.1$. Observer inclination $75°$.



BH non-dimensional spin $\chi = 0.998$

Figure : Black hole with $M = 10M_\odot$, $\dot{M} = 0.1M_{\text{Edd}}$. Novikov–Thorne disk with $\alpha = 0.1$. Observer inclination $75°$.



Section 4

Conclusions & future work

Conclusions

- X-ray polarization data important for compact object studies
- Observational X-ray polarization capabilities likely to increase
- Currently lack of ray-tracing codes that: are free, do polarized radiative transfer, support arbitrary metrics and are well-documented and easy to use
- ARCMANCER fits the bill