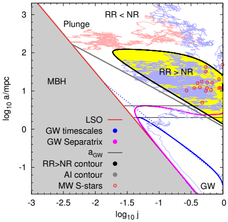
Relativistic loss-cone dynamics

Infall and inspiral rates and branching ratios

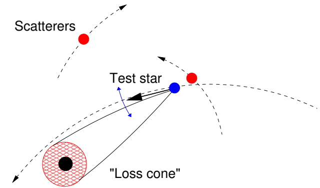
Tal Alenxander

The Weizmann Institute



The stellar dynamical loss-cone problem:

How do stars in a galactic nucleus interact *strongly* with a massive black hole (MBH) and/or fall into it, and at what rates?



Implications

Plunge (infall) processes:
Tidal disruption flares[1,2], tidal detonation[3], tidal scattering[4], gravitational wave (GW) flares

Inspiral processes:
GW extreme mass ratio inspirals (EMRIs)[1,2,5], tidal squeezars[6], accretion disk capture

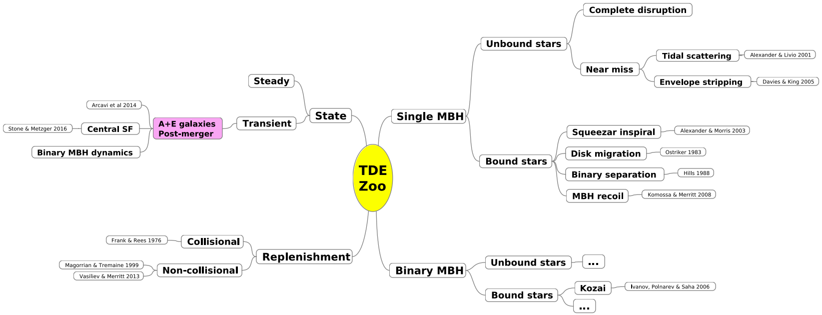
Exotic stellar populations near MBHs[7,8,9]

MBH+stars formation and evolution[10,11,12,13]

How do galactic nuclei randomize?

1:TA & Hopman 2003 2:Bar-Or & TA 2015 3: TA 2005 4:TA & Livio 2001 5:Hopman & TA 2005, 2006a, 2006b 6:TA & Morris 2003 7:TA 1999 8:TA & Livio 2004 9:Perets, Hopman & TA 2007 10:TA & Hopman 2009 11:TA & Kumar 2001 12:Bar-Or & TA 2014 13:Bregman & TA 2012

The (theoretical) TDE zoo

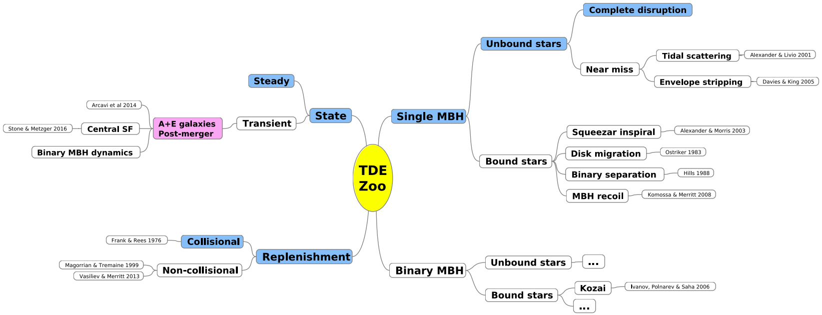


Theory: From initial and boundary conditions to predictions.

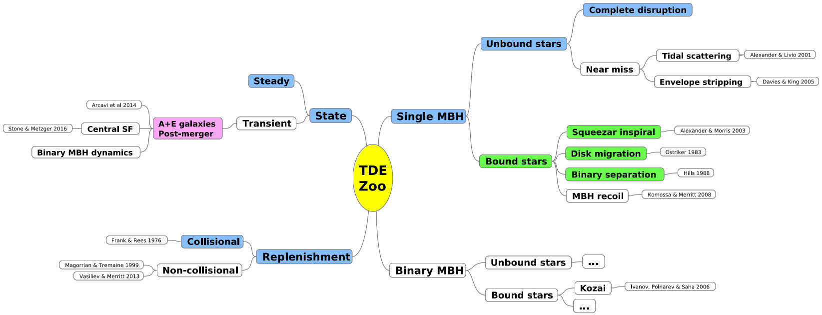
Observations: Provide initial and boundary conditions.

All predictions are conditional.

The (theoretical) TDE zoo



The (theoretical) TDE zoo

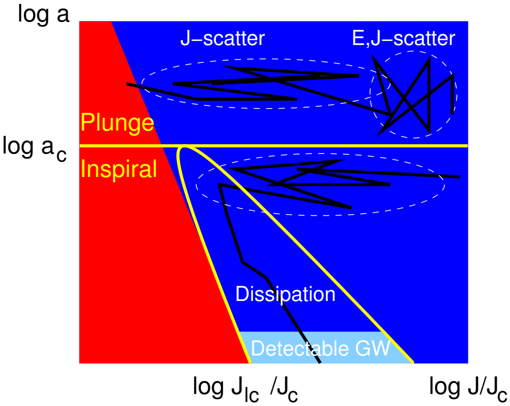


Outline

- Introduction: How do stars get to the MBH?
  - The general loss-cone problem: plunge vs inspiral
  - Relaxation mechanisms: Incoherent vs coherent (resonant)
  - Resonant relaxation: relevance and surprises
- The η-formalism for relativistic loss-cone dynamics (Bar-Or & TA 2014)
  - The stellar background as a correlated noise
  - The role of adiabatic invariance
  - The $N_\star \gg 1$ limit with effective diffusion
- Applications of the η-formalism
  - Cosmic rates of tidal flares and gravity waves from MBHs
  - Young stars near the Milky Way's MBH

The incoherent ("classical") loss-cone: Plunge vs inspiral

Loss primarily by by J-relaxation:  $T_J \sim [J/J_c(E)]^2 \, T_E$



- $\Gamma_{plunge} \sim \dfrac{N_\star(<a_h)}{\langle \log(J_c/J_{lc}) T_E \rangle}$
- $\Gamma_{\mathrm{inspiral}} \sim \dfrac{N_\star(<a_c)}{\langle \log(J_c/J_{lc}) T_E \rangle}$
- $a_c \ll a_h$
- $\Gamma_{\mathrm{inspiral}} \sim O(0.01)\Gamma_{\mathrm{plunge}}$

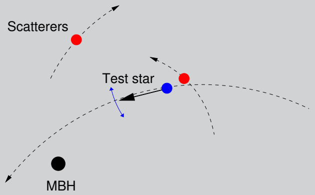
(Lightman & Shapiro 1977; Cohn & Kulsrud 1978)

(TA & Hopman 2003; Hopman & TA 2005)

Relaxation near a MBH

Non-coherent relaxation (NR: E, J)

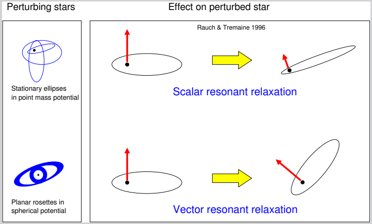
Point-point interactions



$$T_{NR} \sim [Q^2 P / N_\star] / \log Q$$

Resonant relaxation (RR: J)

Orbit-orbit interactions (extended objects)



$$T_{RR} \sim [Q^2 P / N_\star](P / t_{\mathrm{coh}})$$

$$Q = M_\bullet / M_\star$$

Near MBH: $T_{RR} / T_{NR} \sim \log Q (P / t_{\mathrm{coh}}) \ll 1$

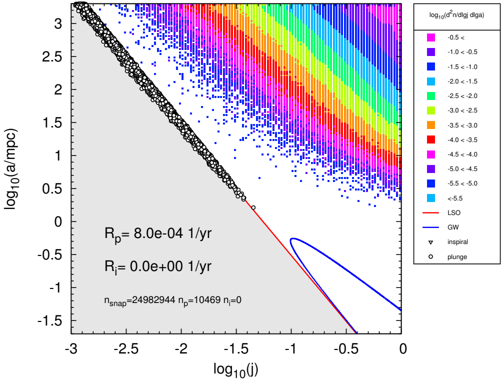
Fast evolution to $J \to 0$: Strong interaction with the MBH

The "danger" of unquenched RR: No inner cusp

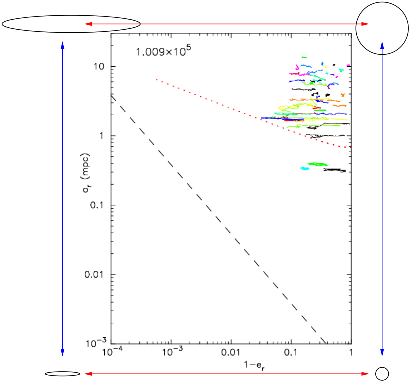
(No gravitational wave EMRIs, no stars, pulsars on GR orbits, no...)

The "fortunate coincidence" conjecture:
(Hopman & TA 2006)

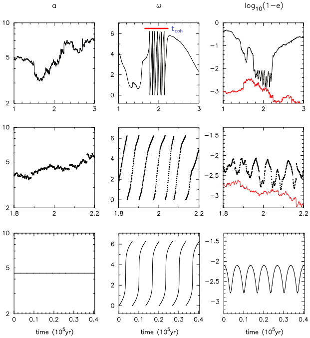
- Unquenched, RR drives all stars to plunge orbits (no EMRIs!).
- $\mathcal{O}(\beta^2 j^{-2})$ GR in-plane Schwarzschild precession becomes significant before $\mathcal{O}(\beta^{5/2} j^{-7} Q^{-1})$ GW dissipation.
- GR precession quenches RR and allows EMRIs to proceed unperturbed, decoupled from the background stars.



First full PN 2.5 *N*-body simulations
An unexpected result: The Schwarzschild Barrier (SB)



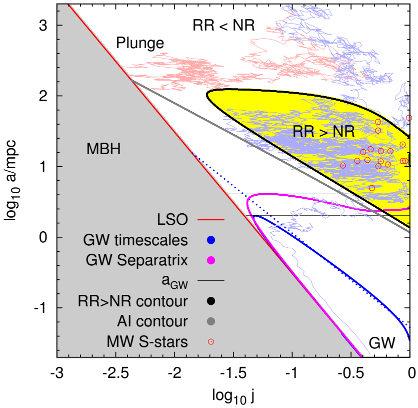
(Merritt, TA, Mikkola & Will 2011)



A reflecting barrier in phase space?

GR precession under an external fixed force?

The relativistic loss-cone with the η-formalism



The η-formalism

Stellar dynamics in the presence of correlated noise

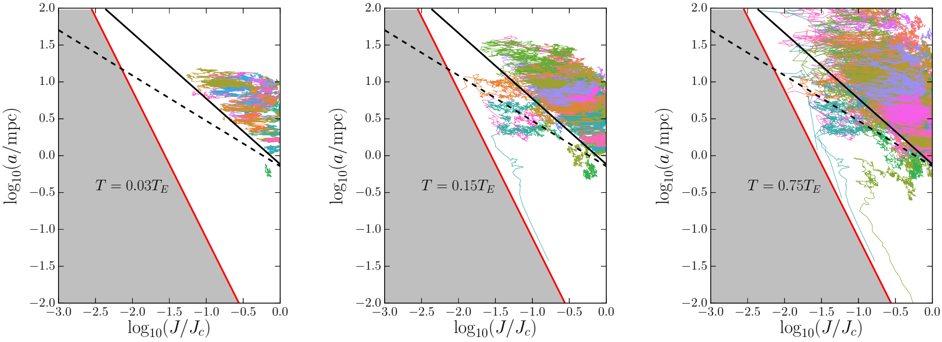
(Bar-Or & TA 2014, 2016)

- Adiabatic invariance[(⋆)] quenches RR at low-j
  - Smoothness of stellar background noise key parameter
- NR dominates evolution on long time scales
- Dynamical modeling of the relativistic loss-cone:
  - Stochastic eq's of motion for evolving test star
  - Effective diffusion for evolving distribution function

A powerful scalable tool for modeling long-term dynamics and loss-rates of galactic nuclei in the realistic $N_\star \gg 1$ limit with Monte Carlo simulations.

⋆ Correct form and interpretation of the "Schwarzschild Barrier" (Merritt, TA, Mikkola & Will 2011).

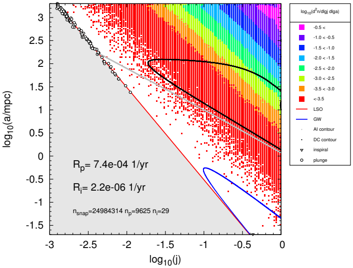
The nature of the phase space barrier (SB=AI)

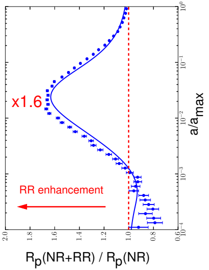


[Diffusive MC realization of “barrier bounces” and “jumps” on short timescales ▶]

(Bar-Or & TA 2016)

Steady state plunge and inspiral rates (single mass models)

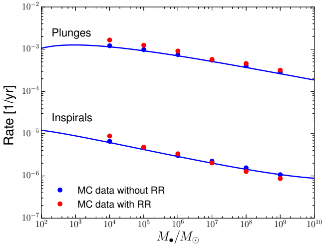




MW galactic nucleus model ("MWEG")

- GR+mass precession
- GW+NR+RR
- Smooth noise+mass quenching

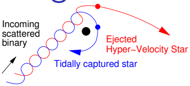
- Adiabatic invariance saves EMRIs: "Fortunate coincidence" validated.

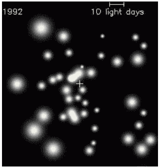


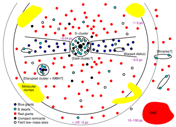
The to do list

- Physical consistency
  - Full statistical characterization of the stellar background noise.
  - Effect of mass-precession on the background.
  - Anomalous J-diffusion? (E-diffusion is, Bar-Or & TA 2013).
- Astrophysical realism
  - Multi-mass stellar populations.
  - In situ star formation.

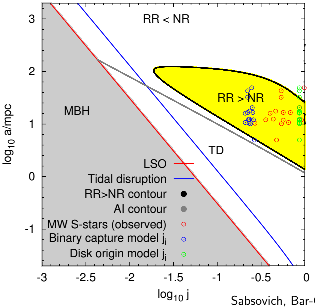
Puzzles of the Galactic center:
Origin of the S-stars? The “missing” old stellar cusp?

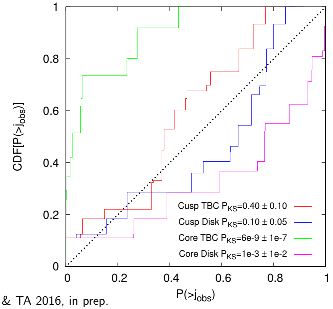






Disk formation and migration





Sabsovich, Bar-Or & TA 2016, in prep.

Conclusions:
1. RR is essential for S-stars post-capture/migration randomization.
2. Best fit model: Tidal capture in dense old cusp of stellar remnants.
3. 5% of captured stars are tidally disrupted, 5% squeezars.

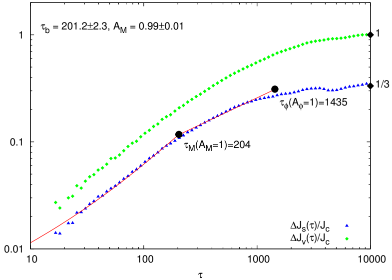
Summary

- General conclusions
  - NR, RR, GW dissipation and secular precession can be treated analytically as effective diffusion with correlated noise.
  - FP/MC models of relativistic loss-cone for galactic nuclei ($N_\star \gg 1$).
  - Plunge / EMRI rates and branching ratios.
  - Quantitative models for "exotic" processes: The Schwarzschild Barrier, Milky Way S-stars, relativistic stars, pulsars, ...
  - RR important close to MBH, on shorter timescales.
  - The steady state depends mostly on NR, which erases AI.
- Effect on TDE rates
  - Only $\mathcal{O}(1)$ enhancement of TDE rates of unbound stars.
  - Relevant for TDEs from bound stars ("exotic channels").

Time-correlated stellar noise

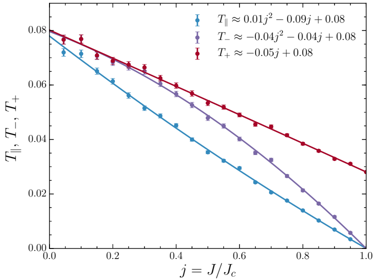
Empirically calibrated torque-noise amplitudes and coherence times

(Eilon, Kupi & TA 2009)



N-body simulations

(Bar-Or & TA 2015)



Static wire simulations

- Time coherence decays fast: $t_{\rm coh}/\Delta t \ll J_c/\Delta J$
- Time-correlated noise vector: $\tau_N(a,j)\boldsymbol{\eta}(t)$

Hamiltonian dynamics with correlated noise (RR v2.0: Bar-Or & TA 2014)

- Canonical action-angle coordinates (Delaunay)

$$(J_z = J\cos\theta, \phi),\ \ (J, \psi),\ \ (J_c = \sqrt{GM_\bullet a}, \omega)$$

- Single star phase-averaged Hamiltonian ($\ell = 1$)

$$H(a, J, J_z, \phi, \psi, t) = H_K(a) + H^1_{GR}(a, J) + H^1_\eta(a, J, J_z, \phi, \psi, t)$$

- Keplerian Hamiltonian

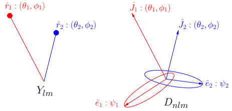
$$H_K = -J_c \nu_r(a)/2$$

- GR Hamiltonian ($\mathcal{O}(\beta^2)$ Schwarzschild (in-plane) precession)

$$H^1_{GR} = -(J_c^2/J)\nu_{GR}(a)\,, \quad \nu_{GR} = 3(r_g/a)\nu_r$$

- Time-correlated Gaussian stellar background ($t_{coh} \equiv \int_0^\infty C(t)\mathrm{d}t$ with $C(0) = 1$)

$$H^1_\eta = \underbrace{\left[\langle H^0\rangle + H^0(t)\right]}_{\text{mass monopole}} + \hat{e}_\psi \cdot \underbrace{\sqrt{2C^1(0)}\boldsymbol{\eta}(t)}_{\text{random vector}}$$



The η-formalism: Stochastic equations of motion

- A stellar noise model $\boldsymbol{\eta}(t)$: A 2-point time correlated random vector in L-space.
- Stochastic EOMs to evolve test star (1st order $\ell = 1$ phase-averaged Hamiltonian):

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{\nu}_{\tau,x}\left(j,\psi,\cos\theta,\phi\right)\cdot\boldsymbol{\eta}\left(t\right), \\ \dot{\psi} &= \boldsymbol{\nu}_{\tau,\psi}\left(j,\psi,\cos\theta,\phi\right)\cdot\boldsymbol{\eta}\left(t\right)+\nu_p\left(j\right) . \end{aligned}$$

where $x = (j, \phi, \cos\theta)$, $\boldsymbol{\nu}_\tau$ = RR torque rates

Coupled Langevin equation for j-trajectory $\varphi(j,t) = \delta(j - j(t))$:

$$\frac{\partial}{\partial t}P(j,t) = -\frac{\partial}{\partial j}\left[\underbrace{\left\{\nu_j(j,\psi,\cos\theta,\phi)\cdot\eta(t)\right\}}_{\text{velocity}}\varphi(j,t)\right]$$

The η-formalism: Effective Fokker-Planck equation

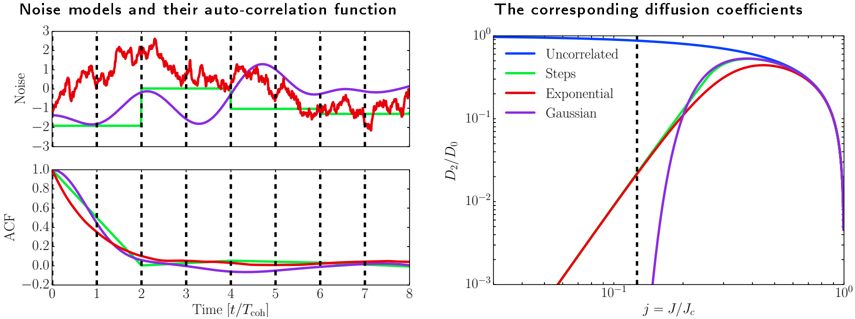
- Evolution of probability density $P(j,t) = \langle \varphi(j,t) \rangle_\eta$:

$$\frac{\partial}{\partial t} P(j,t) = -\frac{\partial}{\partial j} \left\langle \left\{ \nu_j(j,\psi,\cos\theta,\phi) \cdot \eta(t) \right\} \varphi(j,t) \right\rangle_\eta$$

- Outline of derivation:
  - Use Novikov's theorem for Gaussian noise to express $\langle \cdots \rangle$ as integral over functional gradients of histories, via response functions.
  - Use $\nu_j \ll t_{\rm coh}^{-1}$ to neglect ν_j^2 terms and to approximate as constant j, ϕ, u (but not precessing ψ!) in response functions.
- Effective diffusion to evolve probability density (Fokker-Planck equation for j):

$$\begin{aligned} \frac{\partial}{\partial t} P(j,t) &= \frac{1}{2}\frac{\partial}{\partial j}\left\{ j D_2(j) \frac{\partial}{\partial j}\left[\frac{1}{j} P(j,t)\right]\right\}, \\ D_2(j) &= |\boldsymbol{\nu}_{\tau,j}|^2 \mathcal{F}_{C(t)}[\nu_p(j)], \qquad (\mathcal{F}_{C(t)}: \text{ Fourier transform of } \boldsymbol{\eta} \text{ ACF}) \\ D_1(j) &= \frac{1}{2j}\frac{\partial}{\partial j}(j D_2). \end{aligned}$$

Background stellar noise and diffusion



Key issue:

Does the background noise power have a high frequency cutoff ν_0?

Stars with $\nu_{GR}(j) > \nu_0$ ($j < j_0$) decouple from the background stars.

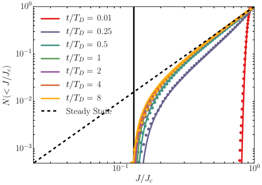
The adiabatic invariance threshold ("barrier"): $j_0(a) \simeq \sqrt{t_{coh}\nu_{GR}(a, j=0)/2\pi}$

The physical noise is expected to be smooth. Smooth noise has a cutoff.

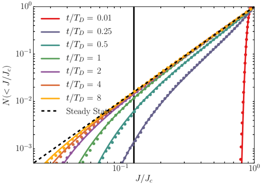
Noise smoothness and adiabatic invariance

Adiabatic invariance: Fast oscillations protect against slow variations

Smooth noise (Gaussian ACF)



Non-smooth noise (exponential ACF)



But, 2-body relaxation erases the SB (The SB is relevant on short timescales only).

*Not a real reflective barrier: Max. entropy is reached, but in exponentially long time (i.e. never).

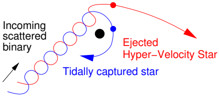
A4: Binary tidal capture (3-body exchange)

Incoming binary:

Mass: $M_{\rm bin} = M_{\rm cap} + M_{\rm HVS}$ (assume $M_{\rm cap} = M_{\rm HVS}$)
sma: $a_{\rm bin}$, Orbital energy relative to MBH: $dE \simeq 0$

Tidal disruption:

$r_t(bin) \simeq a_{\rm bin}(M_\bullet/M_{\rm bin})^{1/3} = r_p(cap) = a_{\rm cap}(1-e_{\rm cap})$
$dE \sim F_t r_t \sim [(GM_\bullet M_{\rm bin}/r_t^3)a_{\rm bin}]r_t \sim GM_\bullet^{1/3}M_{\rm bin}^{5/3}/a_{\rm bin}$



Hills 1988; Gould & Quillen 2003

Captured star initial orbit:

$a_{\rm cap} = -GM_\bullet M_{\rm cap}/2(-dE) \sim (M_\bullet/M_{\rm bin})^{2/3} a_{\rm bin}$
$e_{\rm cap} = 1 - r_p/a_{\rm cap} \sim 1 - (M_{\rm bin}/M_\bullet)^{1/3} > 0.95$ in GC

Conclusions:
1. $e_{\rm cap}$ is independent of binary sma.
2. $a_{\rm cap}$ depends on binary sma and scattering rate's sma dependence.