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University of Nova Gorica
Graduate school, Physics

Turbulence modeling and its applications

Seminar

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Ajdovščina, 11th January 2010

Outline

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Introduction

Laminar flow

- streamlines are smooth and regular,

Turbulent flow

- streamlines break up,
- fluid elements move in a random, irregular and torous fashion.

Turbulence modeling:

- mathematical model that approximates the physical behavior of turbulent flow.
- much simpler than the full time dependent Navier-Stokes equations,
- complex enough to capture the essence of the relevant physics.

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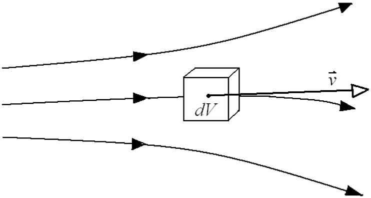
Infinitesimally small moving fluid element of fixed mass.



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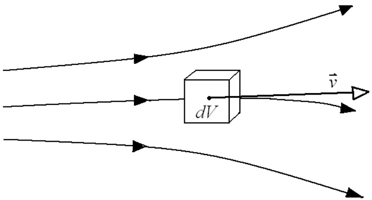

Newton's second law

$$\frac{d(m\,\mathbf{v})}{dt} = \mathbf{F}$$

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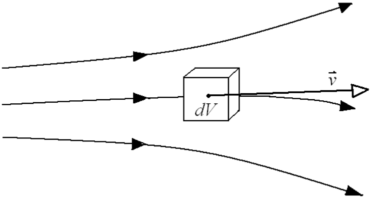

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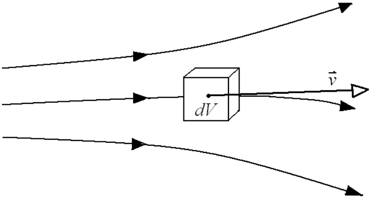

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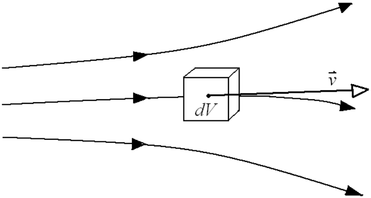

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Viscous stress tensor

$$t_{ij} = 2\mu s_{ij}$$

Strain-rate tensor

$$s_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

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Unsteady, compressible, threedimensional viscous flow.

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Mass conservation equation

$$\frac{\partial u_i}{\partial x_i} = 0$$

Incompressible flow.

Statistical approach

Reynolds 1895

Instantaneous velocity

$$u_i(\mathbf{x}, t) = U_i(\mathbf{x}, t) + u_i'(\mathbf{x}, t)$$

Reynolds time average

$$U_i(\mathbf{x}, t) = \frac{1}{T} \int_t^{t+T} u_i(\mathbf{x}, t) dt$$

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Time averaged equations of motion

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$$\rho \frac{\partial U_i}{\partial t} + \rho U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(2\mu S_{ij} - \overline{\rho u_j' u_i'} \right)$$

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Reynolds-stress tensor

$$\tau_{ij} = -\overline{\rho u_j' u_i'}$$

Fundamental problem of turbulence modeling.

Outline

Algebraic models - zero equation models

- Equation for conservation of mass
- Navier-Stokes equation

$\Rightarrow$ Zero additional equations

- Boussinesq eddy-viscosity approximation (1877)

molecular gradient diffusion process $\approx$ turbulent stress

$$t_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \approx \tau_{ij} = \mu_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

μ ... molecular viscosity
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Boussinesq eddy-viscosity approximation.

Eddy viscosity

$$\mu_T = \rho l_{mix}^2 \left| S_{ij} \right|$$

Prandtl's (1925) mixing length hypothesis.

l_{mix} ... mixing length

Turbulence energy equation models

- conservation of mass and momentum
- n additional differential transport equations
- Boussinesq eddy viscosity approximation: $\tau_{ij} = \mu_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$

Prandtl (1945): eddy viscosity depends upon the kinetic energy of the turbulent fluctuations, k.

Turbulence kinetic energy

$$k = \frac{1}{2}\overline{u_i' u_i'}$$

Eddy viscosity

$$\mu_T = constant \cdot \rho k^{1/2} l$$

Trace of Reynolds stress tensor

$$\tau_{ii} = -\rho \overline{u_i' u_i'} = 2\rho k$$

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$$\rho \frac{\partial k}{\partial t} + \rho U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \rho \epsilon + \frac{\partial}{\partial x_j} \left[(\mu + \mu_T / \sigma_k) \frac{\partial k}{\partial x_j} \right]$$

Turbulence kinetic energy dissipation per unit mass

$$\epsilon = \nu \overline{\frac{\partial u_i'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}}$$

Reynolds stress tensor

$$\tau_{ij} = 2 \mu_T S_{ij} - \frac{2}{3} \rho k \delta_{ij}$$

One equation models

Turbulence kinetic energy dissipation

$$\epsilon \propto k^{3/2}/l$$

Prandtl

Eddy viscosity

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l - the only unspecified part of the one equation model

One equation models

Turbulence kinetic energy dissipation

$$\epsilon \propto k^{3/2}/l$$

Prandtl

Eddy viscosity

$$\mu_T = \rho k^{1/2} l$$

l - the only unspecified part of the one equation model

Spalart and Allmaras (1992) - model equations for the eddy viscosity.

Eddy viscosity

$$\mu_T = \mu_T(\nu, \tilde{\nu})$$

Two equation models

- Transport equation for turbulence kinetic energy, k.
- Transport equation for dissipation of turbulence kinetic energy, ϵ.

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Launder and Sharma (1974) - Standard $k - \epsilon$ model:

Eddy viscosity

$$\mu_T \propto \rho k^2 / \epsilon$$

Turbulence length scale

$$l \propto k^{3/2} / \epsilon$$

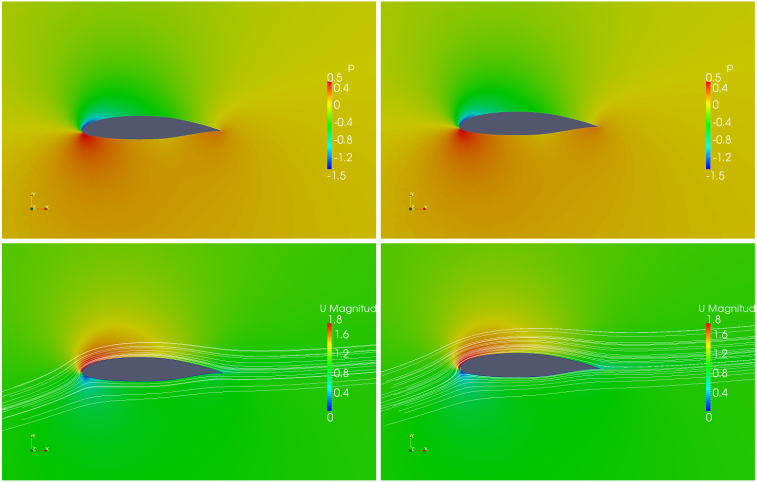
Outline

Simulations

- OpenFOAM - CFD
- simpleFoam - steady, incompressible, viscous, threedimensional flow
- 2D
- $Re = 2.0 \cdot 10^6$

Airfoil - pressure, velocity magnitude and streamlines

Spalart-Allmaras model | $k-\epsilon$ model

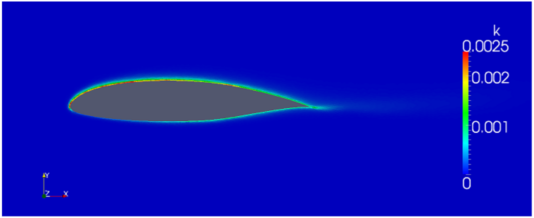


Re=$2.0\cdot 10^6$, $\alpha = 10^\circ$

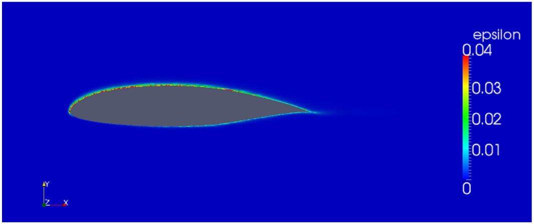
Airfoil - k and ϵ

$k - \epsilon$ model

Turbulence kinetic energy



Dissipation of turbulence kinetic energy



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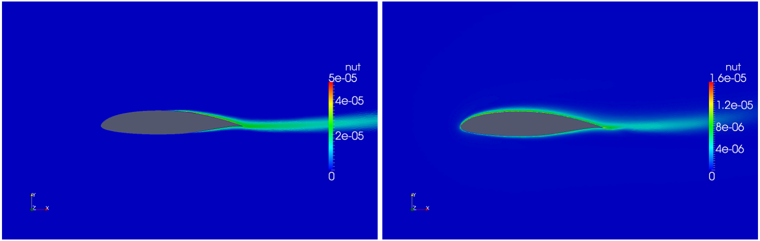
Airfoil - eddy viscosity ν_T

Spalart-Allmaras model $\qquad$ $k - \epsilon$ model

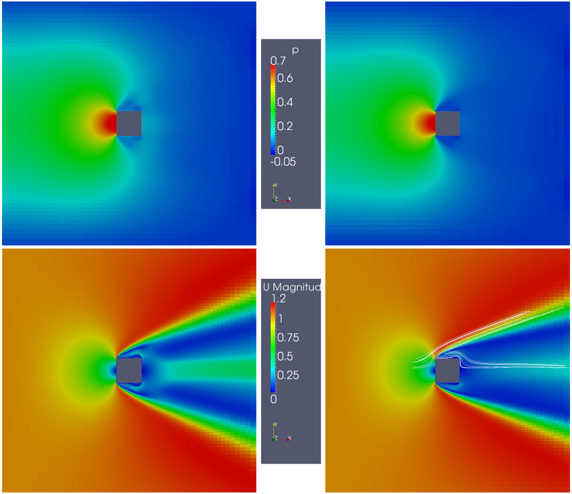


Re=$2.0 \cdot 10^6$, $\alpha = 10°$

Square - pressure, velocity magnitude and streamlines

Spalart-Allmaras $\qquad$ $k-\epsilon$ model

Re=2.0$\cdot 10^6$, $\alpha = 0°$



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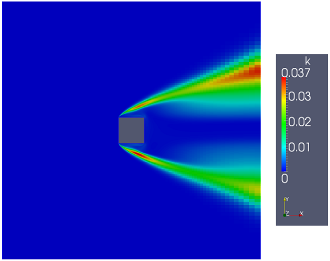

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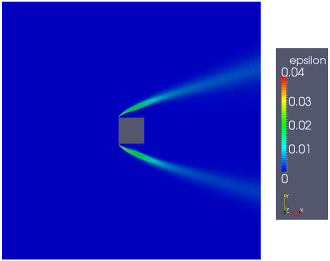

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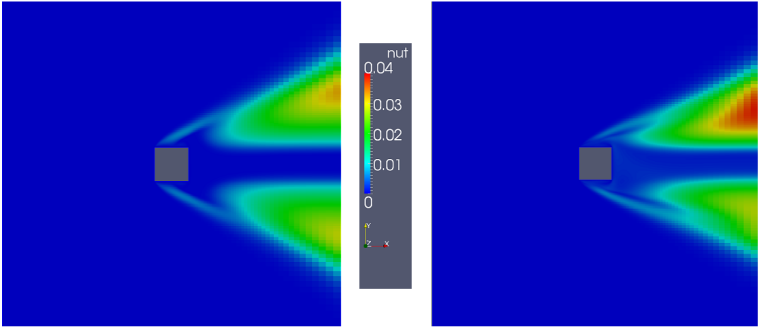

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Conclusions

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